Currently condition method LS is rarely used by surveyors so it is briefly dealt with in chapter 8 of Monograph 13. In the past with limited computer power (or none at all) the condition method was more popular than now. Perhaps it will become useful again in the future.

The first two of these examples are based on my class notes from when I was a student. So my teachers are the original authors. I have added comments and changed the symbols to match those used in Monograph 13. I think the data is not based on real observations, but I may change that to real student observations sometime in the future.

You will see that these example problems do not have to concern themselves with datums, i.e. what to hold fixed, because they adjust the observations only – not the parameters. So these solutions might be comparable to Free Net adjustments.

1. Condition Method Equations

This method includes only the observations, and not parameters, in the model. Conditions, such as misclose sums, are used to adjust the observations.

Arrange the basic equations in the form $F(L) = 0$

where $L$ = adjusted observations and $l$ are the observations (a vector $n$ terms)

$$B = \frac{\partial F}{\partial l} = \begin{pmatrix}
\frac{\partial F_1}{\partial l_1} & \frac{\partial F_1}{\partial l_2} & \frac{\partial F_1}{\partial l_3} \\
\frac{\partial F_2}{\partial l_1} & \frac{\partial F_2}{\partial l_2} & \frac{\partial F_2}{\partial l_3} \\
\frac{\partial F_3}{\partial l_1} & \frac{\partial F_3}{\partial l_2} & \frac{\partial F_3}{\partial l_3} \\
\vdots & \vdots & \vdots \\
\end{pmatrix}$$

$B$ is $n_u$ by $n$

The number of model equations equals $n-u$ (the number of observations minus the number of unknown parameters), and make sure that the model equations are independent.
Model
Functions
misclose (OMC) vector, \( n_e \) terms
Estimated Variance Factor
Residuals
Adjusted observations
Quality of corrected observations
Quality of residuals
Redundancy Numbers

\[ \text{Model} \quad Bv - b = 0 \]
\[ \text{Functions} \quad F(L) = 0 \]
\[ \text{misclose (OMC) vector, } n_e \text{ terms} \quad b = - F(\ell) \]
\[ \text{Estimated Variance Factor} \quad VF = v^T P_v / n_e \]
\[ \text{Residuals} \quad v = Q B^T (B Q B^T)^{-1} b \]
\[ \text{Adjusted observations} \quad L = \ell + v \]
\[ \text{Quality of corrected observations} \quad Q_L = Q - Q B^T (B Q B^T)^{-1} B Q \quad (n \times n) \]
\[ \text{Quality of residuals} \quad Q_v = Q B^T (B Q B^T)^{-1} B Q \quad (n \times n) \]
\[ \text{Redundancy Numbers} \quad r_i = s_v^2 / s^2 = 1 - (s_L^2 / s^2) \]

\( n \) \quad \text{number of observations}
\( n_c \) \quad \text{number of condition equations}
\( n_e \) \quad \text{number of model equations}
\( Q \) \quad \text{a priori VCV matrix of observations} \((n \times n)\)

2. Level net example

Figure 1 Fictitious height network plan

The measurements are assumed to be not correlated. The measurements are:

<table>
<thead>
<tr>
<th>Level line (see fig)</th>
<th>( \Delta H ) (m)</th>
<th>( s^2 ) (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.16</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>12.57</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>6.41</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>1.09</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>11.58</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>5.07</td>
<td>4</td>
</tr>
</tbody>
</table>

Basic equation: \( H_B - H_A - \Delta H_{AB} = 0 \)
Observations: \( \Delta H_{AC} \quad \Delta H_{AD} \quad \Delta H_{CD} \quad \Delta H_{AB} \quad \Delta H_{BD} \quad \Delta H_{BC} \)

Possible condition equations are the loop miscloses, e.g. the three inner loops ABD, BCD, ADC and the outer loop ABC:
\[ \Delta H_{AB} + \Delta H_{BD} - \Delta H_{AD} = 0 \]

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\[ \Delta H_{BC} + \Delta H_{CD} - \Delta H_{BD} = 0 \]
\[ \Delta H_{AD} - \Delta H_{CD} - \Delta H_{AC} = 0 \]
\[ \Delta H_{AB} + \Delta H_{BC} - \Delta H_{AC} = 0 \]

There are 6 observations (n) and 3 unknowns (u). So \( n-u = 3 \), thus we need 3 condition equations. The above four equations are not independent because the last one is simply the sum of the first three. So I select the first three equations as the conditions.

I have chosen units of cm for residuals etc because they are similar to the size of the standard deviation. The VF is always unit less so set the units of P to cancel the units of \( v^2 \). For the diagonal elements of P the units are cm\(^{-2}\) and the units of Q are cm\(^2\). A check equation in the condition method is \( Bv - b = 0 \), so set the units of B to balance the units in this equation, \( b \) is in cm therefore \( Bv \) is in cm. Now all six terms of v are in cm, therefore the units of B must be unit less, and they are.

Substituting observations into the 3 condition equations yield \( b = -F(\ell) \):

<table>
<thead>
<tr>
<th>Condition Equation</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta H_{AB} + \Delta H_{BD} - \Delta H_{AD} = 0 )</td>
<td>-10.0</td>
</tr>
<tr>
<td>( \Delta H_{BC} + \Delta H_{CD} - \Delta H_{BD} = 0 )</td>
<td>10.0</td>
</tr>
<tr>
<td>( \Delta H_{AD} - \Delta H_{CD} - \Delta H_{AC} = 0 )</td>
<td>0.0</td>
</tr>
</tbody>
</table>

With the observations in the order 1 to 6 as listed above the terms in B and Q are:

\[
B = \frac{\partial f}{\partial \ell} = \begin{pmatrix}
0 & -1 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & -1 & 1 \\
-1 & 1 & -1 & 0 & 0 & 0
\end{pmatrix}
\quad \text{and} \quad
Q = \begin{pmatrix}
4 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 4
\end{pmatrix}
\]

An intermediate step, to check your calculations if you are working on this problem yourself.

\[
(BQB^T)^{-1} = \begin{pmatrix}
0.15 & 0.05 & 0.05 \\
0.05 & 0.15 & 0.05 \\
0.05 & 0.05 & 0.15
\end{pmatrix}
\]

Residuals \( v = QB^T (BQB^T)^{-1} b = (0, 2, 2, -4, -4, 4)^T \) cm
[Includes inversion of only a 3x3 matrix]

The adjusted observations are \( \ell + v \):

<table>
<thead>
<tr>
<th></th>
<th>( \Delta H_{AC} )</th>
<th>6.16</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \Delta H_{AD} )</td>
<td>12.59</td>
</tr>
<tr>
<td>2</td>
<td>( \Delta H_{CD} )</td>
<td>6.43</td>
</tr>
<tr>
<td>3</td>
<td>( \Delta H_{AB} )</td>
<td>1.05</td>
</tr>
<tr>
<td>4</td>
<td>( \Delta H_{BD} )</td>
<td>11.64</td>
</tr>
<tr>
<td>5</td>
<td>( \Delta H_{BC} )</td>
<td>5.11</td>
</tr>
</tbody>
</table>

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After some matrix calculations the Quality of corrected observations \( Q_L = Q - QB^T(BQB^{-1})^TBQ \), then standard deviations of corrected observations \( s_L \) (sqrt of diagonal terms of \( Q_L \)) and redundancy numbers \( r = 1 - (s^2/s^2) \) are:

<table>
<thead>
<tr>
<th>( Q_L )</th>
<th>( s_L )</th>
<th>( r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.60</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>0.80</td>
<td>1.20</td>
<td>0.80</td>
</tr>
<tr>
<td>-0.80</td>
<td>0.40</td>
<td>1.20</td>
</tr>
<tr>
<td>0.80</td>
<td>0.80</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.40</td>
<td>-0.80</td>
</tr>
<tr>
<td>0.80</td>
<td>0.00</td>
<td>-0.80</td>
</tr>
</tbody>
</table>

It's a linear problem so there is no need to do a second iteration; except to check the calculations. The second iteration does yield all \( v = 0 \). Substituting the adjusted observations into the original loop misclose equations also gives zero errors.

\[ v^TPv = 20 \]

Three conditions equations so \( VF = 6.7 \) (which is statistically too large)

This concludes the condition method LS adjustment. If you want to determine the heights of the points then you need to do additional calculations. To determine the standard deviations of the adjusted heights you need to do propagation of variance calculations too.

For comparison, a parametric LS solution of this problem was calculated with the height of point A held fixed at 100. The results are the same as the condition method above.

Adjusted Heights:

<table>
<thead>
<tr>
<th>Name</th>
<th>H m</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (1)</td>
<td>100.000</td>
</tr>
<tr>
<td>B (2)</td>
<td>101.050</td>
</tr>
<tr>
<td>C (3)</td>
<td>106.160</td>
</tr>
<tr>
<td>D (4)</td>
<td>112.590</td>
</tr>
</tbody>
</table>

Residuals after adjustment

<table>
<thead>
<tr>
<th>OBS</th>
<th>FROM</th>
<th>TO</th>
<th>MEASUREMENT m cm</th>
<th>STD DEV cm</th>
<th>v</th>
<th>v/s</th>
<th>Redun</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Height Df</td>
<td>1</td>
<td>3</td>
<td>6.16</td>
<td>2.00</td>
<td>0.0</td>
<td>0.0</td>
<td>0.6</td>
</tr>
<tr>
<td>2 Height Df</td>
<td>1</td>
<td>4</td>
<td>12.57</td>
<td>1.41</td>
<td>2.0</td>
<td>1.4</td>
<td>0.4</td>
</tr>
<tr>
<td>3 Height Df</td>
<td>3</td>
<td>4</td>
<td>6.41</td>
<td>1.41</td>
<td>2.0</td>
<td>1.4</td>
<td>0.4</td>
</tr>
<tr>
<td>4 Height Df</td>
<td>1</td>
<td>2</td>
<td>1.09</td>
<td>2.00</td>
<td>-4.0</td>
<td>-2.0</td>
<td>0.6</td>
</tr>
<tr>
<td>5 Height Df</td>
<td>2</td>
<td>4</td>
<td>11.58</td>
<td>1.41</td>
<td>-4.0</td>
<td>-2.8</td>
<td>0.4</td>
</tr>
<tr>
<td>6 Height Df</td>
<td>2</td>
<td>3</td>
<td>5.07</td>
<td>2.00</td>
<td>4.0</td>
<td>2.0</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Variance Factor is 6.67 \([n-u = 3]\)
3. Braced Quad directions example

Theodolite direction observations were made on a braced quadrilateral as shown in figure 3. Solution by the condition method adjusts the directions without need to consider coordinates, datum constraints or orientation parameters, as we see below.

We number the points A as 1, B as 2, C as 3 and D as 4. We assume all directions have the same precision of ± 1" standard deviation. The observations are, in order:

1. DIRECTION     4    2        0 00 00.0
2. DIRECTION     4    1       29 04 04.0
3. DIRECTION     4    3       88 20 53.49
4. DIRECTION     2    1        0 00 00.
5. DIRECTION     2    3       69 42 26.92
6. DIRECTION     2    4      106 45 40.5
7. DIRECTION     1    3        0 00 00.0
8. DIRECTION     1    4       32 31 32.9
9. DIRECTION     1    2       76 41 47.73
10. DIRECTION    3    4      00 00 00.
11. DIRECTION    3    2      54 35 55.62
12. DIRECTION    3    1      88 11 43.2

There are 12 observations (n) and 8 unknowns (u; in the parametric method they would be E and N of C and D and an orientation unknown at each corner). So four (n-u = 4) condition equations are needed.

Calculated Loop Miscloses from the observations are:
Points used:  1  2  4  3  1 =  4.92"  Outer loop
Points used:  1  2  3  1   =  2.23"  Triangle ABC
Points used:  1  2  4  1   = -0.67"  Triangle ABD
Points used:  2  3  4  2   = -2.69"  Triangle BCD
Points used:  1  3  4  1   = -5.59"  Triangle ACD

The outer loop misclose is simply the sum of two triangles so it is not independent of the other equations. Of the other 4 triangle miscloses only three are completely independent, so choose any 3. Here we take the first three triangles listed above and write D12 as the direction AB etc. Remember theodolite directions measure clockwise.
The final condition equation is called a side equation and uses all the directions as though we were completing a circuit around a fictitious point at the centre of the figure. Chapter 3 describes the side condition, for example see fig 3.2 (LHS):

$$\sin A_{11} \times \sin A_{13} \times \sin A_{15} \times \sin A_{17} = \sin A_{12} \times \sin A_{14} \times \sin A_{16} \times \sin A_{18}$$

The side condition is formed by multiples of the sines of angles alternating around the figure as shown by the pattern in the following equation, where D12 is the direction AB.

$$\frac{\sin(D14 - D13)\sin(D23 - D21)\sin(D41 - D42)\sin(D32 - D34)}{\sin(D12 - D14)\sin(D24 - D23)\sin(D43 - D41)\sin(D31 - D32)} - 1 = 0$$

What difference does it make if we use the \( \cdots \times \cdots = 0 \) or \( \cdots / \cdots - 1 = 0 \) ? To work in units of secs throughout these calculations the side equations b term needs to be multiplied by the conversion between radians and seconds i.e. 206264.8… (= 180\*3600/\pi)

Substituting the observations into these 4 condition equations yields the b vector \( b = -F(t) \) in seconds: (-2.23, 0.67, -2.69, 13.45)

The partial derivations in the B matrix for the first three condition equations with the observations in the order listed above are quite simple and unit less:

$$B = \begin{pmatrix} 0 & 0 & 0 & -1 & 1 & 0 & -1 & 0 & 1 & 0 & -1 & 1 \\ -1 & 1 & 0 & -1 & 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 & -1 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

Now to the derivatives for the side equation. Let

\[
\frac{\partial S}{\partial D42} = -\cos(D41 - D42) \times \left( \frac{\sin(D14 - D13)\sin(D23 - D21)\sin(D32 - D34)}{\sin(D12 - D14)\sin(D24 - D23)\sin(D43 - D41)\sin(D31 - D32)} \right) - 1
\]

\[
= -\cot(D41 - D42) \times S = -S / \tan(D41 - D42) = -1.799
\]

This term is unit less, as are all the other terms in B. The terms for all the directions along the sides of the braced quad are similarly derived to the term above. The derivatives for the directions along the diagonals of the braced quad are a little more complicated because those directions appear in two places in the side condition equation. For example, the partial derivative of the side equation with respect D41 is:
\[ \frac{\partial S}{\partial D41} = 5[\cot(D41 - D42) + \cot(D43 - D41)] = 2.393 \]

The last row in B is:

-1.799  2.393  -0.594  -0.370  1.694  -1.324  -1.568  2.597  -1.029  -0.711  2.216  -1.505

Since the directions all have a standard deviation of 1, then Q is the identity matrix.

Residuals, v = QB\textsuperscript{T} (BQB\textsuperscript{T})\textsuperscript{-1} b = (in secs)  [Includes inversion of only a 4x4 matrix] v is a column vector, but I write it here as a row:

-0.49, 1.87, -1.38, -0.05, 0.92, -0.87, 0.25, 0.31, -0.56, 0.81, 0.79, -1.59,

\[ v^\text{T}Pv = 11.51 \text{ and since } n_c = 4 \text{ then } VF = 2.88 \]

The partial derivatives in the first three rows of B are constant and the terms in the last row do not change very much unless there are huge corrections to the direction observations. So B is virtually constant and a second iteration will not normally change the adjusted values, but is useful as a check. A second iteration was performed and all the b terms were 0. That also shows that the loop miscloses using the adjusted directions is now 0. All the residuals were also 0 in the second iteration. So it is a linear adjustment problem.

Notice that we didn’t need to know the coordinates of any points or any distance in the network. We also did not need to hold any coordinates fixed or deal with orientation parameters. But we did need to know how to construct the condition equations particular to this figure.

The condition method does not yield the adjusted coordinates or the error ellipses of the points, they take further calculation.

For comparison, a parametric LS solution of this problem was calculated with coordinates as shown below. The coordinates of A and B are assumed to be known and fixed. Are the results are the same as the condition method above? Yes!

### Adjusted Coordinates after iteration 5

<table>
<thead>
<tr>
<th>Point</th>
<th>Fixed</th>
<th>Name</th>
<th>E (m)</th>
<th>N (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ENH</td>
<td>A</td>
<td>3474.0190</td>
<td>11480.2820</td>
</tr>
<tr>
<td>2</td>
<td>ENH</td>
<td>B</td>
<td>3295.9940</td>
<td>9726.7060</td>
</tr>
<tr>
<td>3</td>
<td>H</td>
<td>C</td>
<td>6297.2453</td>
<td>10502.6243</td>
</tr>
<tr>
<td>4</td>
<td>H</td>
<td>D</td>
<td>5630.4448</td>
<td>8756.9802</td>
</tr>
</tbody>
</table>

### RESIDUALS AFTER ADJUSTMENT (5 iterations) s = 1” for all obs

<table>
<thead>
<tr>
<th>OBS</th>
<th>TYPE</th>
<th>FROM</th>
<th>TO</th>
<th>MEASUREMENT</th>
<th>v</th>
<th>v/s</th>
<th>Redun</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Direction</td>
<td>4</td>
<td>2</td>
<td>00 00.0</td>
<td>-0.5”</td>
<td>-0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>Direction</td>
<td>4</td>
<td>1</td>
<td>29 04 04.0</td>
<td>1.9”</td>
<td>1.9</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>Direction</td>
<td>4</td>
<td>3</td>
<td>88 20 53.5</td>
<td>-1.4”</td>
<td>-1.4</td>
<td>0.3</td>
</tr>
<tr>
<td>4</td>
<td>Direction</td>
<td>2</td>
<td>1</td>
<td>00 00.0</td>
<td>0.0”</td>
<td>0.0</td>
<td>0.3</td>
</tr>
<tr>
<td>5</td>
<td>Direction</td>
<td>2</td>
<td>3</td>
<td>69 42 26.9</td>
<td>0.9”</td>
<td>0.9</td>
<td>0.4</td>
</tr>
<tr>
<td>6</td>
<td>Direction</td>
<td>2</td>
<td>4</td>
<td>106 45 40.5</td>
<td>-0.9”</td>
<td>-0.9</td>
<td>0.3</td>
</tr>
</tbody>
</table>
After iteration 5 the Variance Factor is 2.88 [n-u = 4]

4. Traverse example

Here we solve the traverse given in Chapter 3, by the condition method. We will omit the radiation from 103 to 201. [Once we get the adjusted observations for the rest of the traverse we can work out the bearing for the radiation, and then use it with observed distance to get coordinates. There are no corrections (zero residuals) for radiation observations.]

Figure 4. Student traverse at Morpeth survey camp

I will renumber the points here to make subscripts easier to follow in the equations below. Thus point 19 becomes 9, 1 becomes 7, 1 becomes 8, 101 becomes 1 etc.

The given data, in FIXIT format is:

<table>
<thead>
<tr>
<th>COMMENT</th>
<th>Dist mm</th>
<th>ppm</th>
<th>Dirn &quot;</th>
<th>mm cent</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSD</td>
<td>3.0</td>
<td>0.0</td>
<td>5.0</td>
<td>0.0</td>
</tr>
<tr>
<td>COORDINATE</td>
<td>9 EN RO</td>
<td>132.08</td>
<td>6981.69</td>
<td></td>
</tr>
<tr>
<td>COORDINATE</td>
<td>7 EN START BM</td>
<td>171.94</td>
<td>7339.56</td>
<td></td>
</tr>
<tr>
<td>COORDINATE</td>
<td>8 EN END BM</td>
<td>20.58</td>
<td>7392.58</td>
<td></td>
</tr>
<tr>
<td>DIRECTION</td>
<td>7 9</td>
<td>0 00</td>
<td>00</td>
<td></td>
</tr>
<tr>
<td>DIRECTION</td>
<td>7 1</td>
<td>211 52 52</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DIRECTION</td>
<td>1 7</td>
<td>0 00</td>
<td>00</td>
<td></td>
</tr>
<tr>
<td>DIRECTION</td>
<td>1 2</td>
<td>106 25 56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DIRECTION</td>
<td>2 1</td>
<td>0 00</td>
<td>00</td>
<td></td>
</tr>
</tbody>
</table>

© 2010 B Harvey
DIRECTION  2  3      168 43 16
DIRECTION  3  2      0 00 00
DIRECTION  3  4     101 26 53
DIRECTION  4  3      0 00 00
DIRECTION  4  8     155 43 06
DIRECTION  8  4      0 00 00
DIRECTION  8  9     134 15 33
H DISTANCE  7  1      63.575
H DISTANCE  1  2      72.854
H DISTANCE  2  3      75.832
H DISTANCE  3  4      61.556
H DISTANCE  4  8      84.805

There are 17 observations, we will use them in the order listed above. Traverses of this type are known to have 3 condition equations, namely one for each misclose in angles, in Eastings, and in Northing. So the b and v vectors will have 17 terms and the B matrix will be 3 by 17. Approximate coordinates for the intermediate traverse points are not required. If this was a loop traverse we wouldn’t need any coordinates, and would assign a bearing to one line.

We set the input standard deviation terms ppm and centring to zero to ease the calculations in this example.

From the fixed coordinates we calculate the ‘known’ bearings at the start and end of the traverse, B79 and B89 (I carry the full D M S values on a spreadsheet and show rounded off values in degrees and a decimal here): B79 = 186.4° B89 = 164.8°

Our first condition (CE1) is that the angular misclose for perfectly adjusted data is zero. Using B12 to represent bearing 1 to 2, D12 to represent direction 1 to 2, and d12 to represent distance 1 to 2.

CE1: B79+D71-D79-360+D12-D17+180+D23-D21-180+D34-D32-180+D48-D43-180+D89-D84-180-B89 = 0

Remember that to produce the bearings, we desire an answer between 0 and 360. Also be careful with using angles or directions to calculate bearings, here we assume they are clockwise from the backsight to the foresight.

Inserting the data gives bearings carried forward and the b term = - F(ℓ)

The bearings are:

B71 38.2
B12 324.7
B23 313.4
B34 234.8
B48 210.6
B89 164.8

Angular misclose’s b term = 8.0°

Our second condition (CE2) is the misclose in Easting. This is easier to write in terms of bearings than directions. So we determine ‘raw’ bearings using our directions starting from one end of the traverse, in our case we start at 7, as shown above. But
when differentiating the equation to form the B matrix we need to recall the formula for each bearing. For points in sequence i j k: Bjk = Bij + Djk − Dji − 180°

CE2: E7 + d71sinB71 + d12sinB12 + d23sinB23 + d34sinB34 + d48sinB48 − E8 = 0

Inserting our data in this equation gives 0.0316m, so b = -31.6 mm

Similarly, the third condition is:

CE3: N7 + d71cosB71 + d12cosB12 + d23cosB23 + d34cosB34 + d48cosB48 − N8 = 0

Inserting our data in this equation gives -0.0324 m, so b = 32.4 mm

Thus the b vector in seconds and mm is:

8.0
-31.6
32.4

The three conditions, in the order above, are differentiated by the 17 observations, also in the order above, to produce the terms in the B matrix. A few terms are shown in detail, the others are similarly derived.

The derivatives of CE1 are simple, e.g. \( \frac{\partial CE1}{\partial D71} = 1 \)

The other derivatives are not simple because a direction or angle observation appears in bearings of all subsequent lines around a loop. One of the terms (it appears in following bearings too) in CE2 containing D12 is

\[ \ldots + d12\sin B12 \]

which really is \[ \ldots + d12\sin(B79+D71-D79-360+D12-D17+180) \] so

The equations become longer the further we move along the traverse. The full details of this example are not further included in these notes. See Cooper’s book for an example. Also, see the paper Efficient Least Squares Traverse Adjustment Using Spreadsheets by Hu, Tan and Barnes in the journal Australian Surveyor June 2003.

**Alternative Solution Method using Spreadsheet Solver function.**

It is possible to solve this problem using MS Excel’s Solver function, as shown in the spreadsheet screen capture below. The answers are the same as those obtained by FIXIT3 software using a parametric least squares solution. Note that in this example below we minimise \( \sum (v/s)^2 \), so it is a least squares solution. While the solution uses coordinates it does so only as intermediate steps to work out bearings and coordinate miscloses. The adjustment deals with the residuals v, not the changes in coordinates.
### CONDITION METHOD EXAMPLES

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
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**CONDITION METHOD SOLUTION By Solver**

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<th>m</th>
<th>n</th>
<th>d</th>
<th>v</th>
<th>adj</th>
<th>E</th>
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<td>3.0</td>
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<td></td>
</tr>
</tbody>
</table>

17 How to use solver:
18 Set up initially with all v set to zero.
19 So coordinates are calculated with raw bearings and distances
20 This allows miscooses to be calculated.
21 Get solver to determine the values in all the v cells is by changing values.
22 The constraints are that the misc in angles and misc in E and if all must equal zero, put these in with the Add button
23 Set Target cell so that we minimise vTPv
24 Then press Solve button.
25 As shown in screen capture.