A Doppler Based Navigation Algorithm

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Biography

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Dr. William R. Michalson is an Associate Professor in the ECE Department at WPI, where he also directs the Satellite Navigation Laboratory. The majority of his research focuses on the development, test, and evaluation of GPS integrity monitoring algorithms, with an emphasis on integrity monitoring for sole-means navigation and precision approach. He was previously with Raytheon Company where he developed computer system architectures for space-based data and signal processors.

Abstract

This algorithm uses the GPS satellite system for navigation. It is an innovative algorithm, because it employs only the Doppler measurements and starts with a known initial location of the receiver. Initially, this algorithm was developed for fixed point GPS transmitters. In the paper we have successfully attempted to adapt this algorithm for moving satellites. This algorithm appears to perform far better than the pseudorange conventional least square algorithm (LSA), due to the small standard deviation of the Doppler measurement noise. While we have already developed and tested this algorithm with simulated data, we will attempt to provide test results with live satellites.

Introduction

The Doppler based navigation idea is not new. In the late 50s Guier and Weiffenbach proposed a Doppler Satellite Navigation system, which could provide navigation accuracy of about one-half mile provided that proper use of the full Doppler measurement is made [1]. At that time both the navigation scientists and engineers were faced with challenge of designing an end-to-end system, which as we know is one of the predecessors of the GPS system. About the same time Guier and Weiffenbach proposed a technique for measuring the Doppler shift of radio
transmissions from satellites, thus enabling navigation based on their algorithm [1-2]. Applications of Doppler measurements were also found in relativity, space probe tracking, and geodesy [3].

In the late 1960s, R. R. Newton, trying to persuade the space community, argued that it is feasible to build a system of navigation by measuring the Doppler shift in the radio transmissions from a near-Earth satellite and a simple equipment for conducting the measurements [4]. By considering a particular case, R. R. Newton, described in detail that the calculations required to produce a navigation solution accurate to about 500 m, once the measurements are recorded, can be performed even by hand within 10-15 min [4].

In the early 1970s J. Glish, proposed a closed form solution for a Doppler Satellite Navigation system [5], based on the early work proposed by R. R. Newton [4]. However, J. Glish does not present the expected accuracy of his approach.

The Doppler measurements were also combined with the range measurements to yield an instantaneous positioning with a single satellite [6]. This same idea is used by N. Levanon to produce user terminal position instantly by a two-communication system between a user terminal on the earth surface and a single low earth orbit (LEO) [7].

Since the Doppler measurements are less noisy, an algorithm, which would produce the user's position based only on Doppler, would be very desirable. In [8] we have proposed an alternative algorithm that uses the Doppler measurements for fixed pseudolites.

Since, this approach appears feasible for fixed pseudolites we decided to modify this algorithm for moving satellites, which is explained in more detail in the following section. In the section that follows after the algorithm description, we describe the error sources involved in the Doppler measurements and if possible we discuss ways of mitigating them. The simulation section contains simulation scenarios design to support the validity of the approach. The paper is concluded with conclusions, discussion, and a list of useful references.

**Algorithm Description**

**1. One dimensional case**

Assume that one moving transmitter A (possible a satellite) and one receiver B are positioned on a one-dimensional axis as pictured in figure 1.

We further assume that the receiver B is perfectly synchronized with the transmitter A and that the initial location of the receiver B is known. As in [8], let \( d_{AB}^k \) denote the distance between the moving transmitter A and the moving receiver B at discrete time \( k \). Assume that the receiver B moves towards the transmitter A by the amount of \( \vec{v}_{AB}^k \). This is analytically determined from,

\[
\vec{v}_{AB}^k \approx d_{AB}^{k+1} - d_{AB}^k. \quad (1)
\]

Assuming that the observations are performed at 1 Hz rate, relationship (1) gives the formula for approximating the average velocity during 1 sec interval. Under the above assumptions we can rewrite (1) is accordance with,

\[
\vec{v}_{AB}^k \approx x_B^{k+1} - x_B^k - (x_A^{k+1} - x_A^k). \quad (2)
\]

This expression enables us to form the navigation equation for the one-dimensional case, which can be written as,

\[
x_B^{k+1} \approx x_B^k + \vec{v}_{AB}^k + (x_A^{k+1} - x_A^k). \quad (3)
\]

It is evident in this case that by knowing the Doppler at any epoch we can determine the
location of the receiver B with perfect accuracy in the one-dimensional case.

Note: The same expression can be obtained when the receiver B moves away from transmitter A with the only modification of (3): the plus sign, (+), in front of \( \vec{v}_{AB} \) now becomes minus, (–).

2. Two dimensional case

Assuming an error free environment with perfect synchronization between the transmitters and the receiver, at least two moving transmitters A and B are required to determine the correct location of a single receiver C in a two-dimensional plane (see figure 2).

We will again assume that the two transmitters are moving with the coordinate system under consideration. We will also assume that the initial location of receiver B is known. Similar to the two dimensional case in [8], it can be shown analytically that for moving transmitters A and B and for known distances AC and BC there are two solutions in which C can be located.

![Two-dimensional system diagram](image)

Both points \( C^{k+1} \) and \( C^{k+1} \) are symmetrical with respect to the line that passes between \( A^{k+1} \) and \( B^{k+1} \) as supposed to A and B [8] (see figure 2). Denote that the new ranges between the receiver C and transmitters A and B as \( d_{AC}^{k+1} \) and \( d_{BC}^{k+1} \) respectively. Similar to the one-dimensional case the receiver C moves towards transmitters A and B by the amount of:

\[
\vec{v}_{AC}^k \equiv d_{AC}^{k+1} - d_{AC}^k, \tag{4}
\]

\[
\vec{v}_{BC}^k \equiv d_{BC}^{k+1} - d_{BC}^k. \tag{5}
\]

The new point \( C^{k+1} \) is found from the intersection of two circles—one with center at transmitter \( A^{k+1} \) and with radius given by (6) and the other circle with center at the transmitter \( B^{k+1} \) and with radius given by (7). The analytical expression of these distances can be written as,

\[
d_{AC}^{k+1} \equiv d_{AC}^k + \vec{v}_{AC}^k, \tag{6}
\]

\[
d_{BC}^{k+1} \equiv d_{BC}^k + \vec{v}_{BC}^k. \tag{7}
\]

Without showing all the work the solution for the new location of the receiver C is determined from:

\[
x_C^{k+1} = A y_C^{k+1} + B, \tag{8}
\]

\[
y_C^{k+1} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \tag{9}
\]

where,

\[
A = \frac{y_A^{k+1} - y_B^{k+1}}{x_B^{k+1} - x_A^{k+1}}, \tag{10}
\]

\[
B = \frac{-C}{2(x_B^{k+1} - x_A^{k+1})}, \tag{11}
\]

\[
C = d_{AC}^k - d_{BC}^k + \vec{v}_{AC}^k - \vec{v}_{BC}^k + D, \tag{12}
\]

\[
D = \left( x_B^{k+1} \right)^2 - \left( x_A^{k+1} \right)^2 + \left( y_B^{k+1} \right)^2 - \left( y_A^{k+1} \right)^2, \tag{13}
\]

\[
a = A^2 + 1, \tag{14}
\]

\[
b = A(B - x_A^{k+1}) + y_A^{k+1}, \tag{15}
\]

\[
c = (B - x_A^{k+1})^2 + (y_A^{k+1})^2 - d_{AC}^k - \vec{v}_{AC}^k. \tag{16}
\]
In order to resolve the double location ambiguity we use theorem 1 [8]. Therefore, the correct $d_{CC}^{k+1}$ should be chosen from the two solutions of (9) if,

$$d_{CC}^{k+1} \leq r_c^k \leq d_{CC}^{k+1}.$$  \hspace{1cm} (17)

where,

$r_c$ denotes the radius (equals the sum Doppler measurements, see 17) of the circle inside of which the receiver, C, is found,

$$r_c^k = \overline{v}_{AC}^k + \overline{v}_{BC}^k,$$  \hspace{1cm} (18)

$d_{CC}^{k+1}$ denotes the distance between the current position and the new correct position of receiver by, such as

$$d_{CC}^{k+1} = \sqrt{(x_c^{k+1} - x_c^k)^2 + (y_c^{k+1} - y_c^k)^2},$$  \hspace{1cm} (19)

$d_{CC}^{k+1}$ denotes similarly the distance between the current position of the receiver and the mirror image of the new, correct position of the receiver, in accordance with,

$$d_{CC}^{k+1} = \sqrt{(x_c^{k+1} - x_c^k)^2 + (y_c^{k+1} - y_c^k)^2}.$$  \hspace{1cm} (20)

3. Three dimensional case

In a error free and perfect synchronization environment, the correct location of the moving receiver D in three dimensions can be determined with help of the three moving transmitters (A, B, and C), which are not in the same line at any time as shown in figure 3. Denote that the new ranges between receiver D and transmitters A, B and C are $d_{AD}^{k+1}$, $d_{BD}^{k+1}$, and $d_{CD}^{k+1}$ respectively (see figure 3). Similar to the one-dimensional and two-dimensional cases the receiver D moves towards transmitters A, B, and C by the amount of,

$$\overline{v}_{AD}^k \approx d_{AD}^{k+1} - d_{AD}^k,$$  \hspace{1cm} (21)

$$\overline{v}_{BD}^k \approx d_{BD}^{k+1} - d_{BD}^k,$$  \hspace{1cm} (22)

$$\overline{v}_{CD}^k \approx d_{CD}^{k+1} - d_{CD}^k.$$  \hspace{1cm} (23)

![Figure 3: Three-dimensional system diagram](image)

The new point $D^{k+1}$ is found from the intersection of three spherical surfaces—one with its center at transmitter $A^{k+1}$ and with radius given by (21), one circle with its center at the transmitter $B^{k+1}$ and with radius given by (22), and the final circle with its center at transmitter $C^{k+1}$ and with radius given by (23),

$$d_{AD}^{k+1} \approx d_{AD}^k + \overline{v}_{AD}^k,$$  \hspace{1cm} (24)

$$d_{BD}^{k+1} \approx d_{BD}^k + \overline{v}_{BD}^k,$$  \hspace{1cm} (25)

$$d_{CD}^{k+1} \approx d_{CD}^k + \overline{v}_{CD}^k.$$  \hspace{1cm} (26)

Without showing all the work the solution for the new location of the receiver D is determined from:

$$x_D^{k+1} = d_1 z_D^{k+1} + e_1,$$  \hspace{1cm} (27)

$$y_D^{k+1} = d_2 z_D^{k+1} + e_2,$$  \hspace{1cm} (28)

$$z_D^{k+1} = \frac{-b_3 \pm \sqrt{b_3^2 - a_3 c_3}}{a_3},$$  \hspace{1cm} (29)

where,

$$a_3 = d_1^2 + d_2^2 + 1,$$  \hspace{1cm} (30)

$$b_3 = d_1 e_1 + d_2 e_2 - d_1 z_C^{k+1} - d_2 y_C^{k+1} - z_C^{k+1},$$  \hspace{1cm} (31)
Expression (20) can serve as a criterion for selecting the correct $z_{D}^{k+1}$.

4. Multidimensional noisy (final) case

The expressions for the LSA solution utilizing DD pseudorange obtained for the multiple range sources is similar to that obtained in [8]. We take the time here, however, to explain some important differences of modifying the Doppler based navigation algorithm for satellites.

First we note that the satellites' range vector from the center of the earth can be expressed based on the Taylor series expansion as,

$$R[k + 1] = R[k] + \dot{R}[k] + 0.5\ddot{R}[k] + O(\dddot{R}[k]),$$

where, $R[k]$, $\dot{R}[k]$, $\ddot{R}[k]$, and $\dddot{R}[k]$ are the satellite's range, range rate, the rate of range rate, and third derivative of range respectively, from the center of the earth at the $k^{th}$ epoch. The quantity $O(\dddot{R}[k])$ denotes the remainder of the Taylor series expansion for range derivative terms of orders higher than the third. Expression (49) assumes a 1-Hz data rate. A similar expression can be obtained for the range vector between the moving satellite and the user as,

$$R_U[k + 1] = R_U[k] + \dot{R}_U[k] + 0.5\ddot{R}_U[k] + O(\dddot{R}_U[k]),$$

where $R_U[k]$, $\dot{R}_U[k]$, $\ddot{R}_U[k]$, and $\dddot{R}_U[k]$ are the satellite's range, range rate, the rate of range rate, and third derivative of range respectively, from the user $U$ at the $k^{th}$ epoch. The only observable quantities of expression (50) are the range and the range rate utilizing any commercial receiver in the market. The challenge in this case is to derive a quantity from Doppler measurement imitating the raw pseudorange measurements. In order to accomplish that, we first estimate the range knowing the initial location of the receiver and the location of the satellites. Next, the range rate can be replaced with the Doppler measurements. The rate of the range rate can be approximated with successive differences of Doppler. Finally, a process noise sequence can serve as the remainder of the Taylor series expression $O(\dddot{R}_U[k])$ (see (50)). The result of this work would be the expression for the
Doppler derived pseudorange vector, which looks like,
\[
\rho_U[k+1] = \hat{R}_U[k] + \hat{\phi}_U[k] + 0.5(\hat{\phi}_U[k] - \hat{\phi}_U[k-1]) + w_U[k].
\] (51)

Based on formulation (51) for the Doppler derived pseudorange we can derive an expression for the noise quantities as,
\[
\sigma_\rho = 3\sigma_\phi + \sigma_w. \quad (52)
\]

Table 1: Estimated Doppler measurement noise

<table>
<thead>
<tr>
<th>Item</th>
<th>SAT1</th>
<th>SAT2</th>
<th>SAT3</th>
<th>SAT4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\bar{d}) (cm/s)</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>S (cm/s)</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>11</td>
</tr>
</tbody>
</table>

The expression of Doppler derived pseudorange (51) is then used to compute the user solution for either the conventional or modified LSA [8].

**Error Sources**

The error sources affecting the Doppler measurement can be classified in four categories: ionosphere, troposphere, receiver measurement noise, and multipath. Although extensive study of the ionosphere and troposphere models are done by J. A. Klobuchar [11] and J. J. Spilker Jr. [12], a model which performs correction of both the ionosphere and troposphere effects for a single frequency receiver is yet to come.

The Doppler receiver measurement noise was estimated and the result of this work is shown in Table 1, where \(d\) denotes the sample mean and S and standard deviation.

It is conceivable that the effect of the multipath and ionosphere, troposphere, and multipath are included in the Doppler measurement noise estimate.

**Simulation**

Simulation results are provided for one moving scenario.

The satellite constellation can generated using either almanac data or ephemeris data [10]. However, for simplicity the simulation results were obtained for a GPS constellation driven by almanac data. The moving scenario, depicted in figure 4, was selected from one the indoor geo-location applications [6].

![Figure 4: Satellite, moving receiver layout](image)

- **Raw pseudorange CLSA**

First we process the raw pseudorange using the CLSA and obtain the lateral and vertical position error for pseudorange measurement error (1 sigma) ranging from 0.01 to 5 m (see figures 5 and 6).

- **Raw pseudorange MLSA**

Next we process the raw pseudorange measurements utilizing the modified least square algorithm (see figures 7 and 8).

- **Doppler derived pseudorange CLSA**

Next, we process the Doppler derived pseudorange and obtain the lateral and vertical position error for Doppler measurement error (1 sigma) ranging from 0.01 to 0.1 m (see figures 9 and 10).
Figure 5: Lateral position error vs. pseudorange measurement error

Figure 6: Vertical position error vs. pseudorange measurement error

Figure 7: Lateral position error vs. pseudorange measurement error

Figure 8: Vertical position error vs. pseudorange measurement error

Figure 9: Lateral position error vs. Doppler measurement error

Figure 10: Vertical position error vs. Doppler measurement error

- Doppler derived pseudorange MLSA

Here, we repeat case III only utilizing the modified least square algorithm and the results are presented in figures 11 and 12.
Summary and Discussion

It appears that the CLSA algorithm provides erroneous lateral and vertical position for the user despite the pseudorange or Doppler measurement error. For a constellation of 10 satellites and a system PDOP of closer to 1, obviously the geometry is not the primary deteriorating source of the navigation solution. A detailed discussion of this phenomenon will be the object of another article. We just mention here that it appears from the simulation point of view that the method under observation is incorrect.

On the other hand, the MLSA algorithm appears to provide consistent results with pseudolite data only [8].

Thus, processing Doppler derived pseudorange and utilizing the MLSA algorithm yields a navigation solution 50 times better than the pseudorange MLSA.

References


