An Innovative Navigation Algorithm Using a System of Fixed Pseudolites

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Biography
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Abstract
An innovative navigation algorithm, which utilizes a system of fixed pseudolites is derived, analyzed, discussed, and simulated in this paper. Although this algorithm uses the same Doppler measurements as a conventional least square algorithm LSA (which produces user velocity and user clock drift) or Kalman based filter (which obtains user position/velocity etc.), in essence the Doppler measurements are applied differently and therefore this is the reason why this is an innovative algorithm. The solution accuracy obtained using this algorithm is compared with solution accuracy from the conventional pseudorange LSA. The result of this comparison is also presented in the paper. We will desire to present live test results if they are available at the time of presentation.

Introduction
The Doppler effect, also known as the Doppler shift, is a fundamental concept in astronomy, physics and navigation science [1-3].

In the 1920s, astronomers found something most peculiar about the spectra of stars of distant galaxies [1]. Their spectra were shifted by a constant amount in the red spectrum compared with the spectra of stars of our galaxy. Of course, this implication can be explained easily with the Doppler effect [1], which is the relationship between the signal frequency and the speed of the transmitting and/or receiving device.

When the Doppler effect was neglected, scientists led to erroneous conclusions in early optical physics' conclusions (The experiment of Einstein and Rupp) that were not supported by classical mechanics [2].

In navigation science, during the investigation of Transit system, the Doppler shift data were recorded at one site during a single pass of the Sputnik 1 to determine its entire orbit [3],
assuming a non-changing satellite orbit. Although this technique had world coverage and periodic updates, it had still limited accuracy, required post-processing time, and voluminous equipment from the user's point of view.

Doppler measurements were used to determine the earth gravitational field, which led to accurately determining and predicting the Transit satellite's orbit [3].

The new idea starting with the Timation concept (in 1964) measures range rather than Doppler and to use these measurements for navigation, which is the concept we are most familiar with [3]. A study conducted by the RCA Astro-Electronics Division in Princeton, NJ, seems to suggest that a combination of range and Doppler measurements provides near-instantaneous positioning with only a single satellite [3]. Dual-phase Doppler measurements were also seen as a means of determining the ionospheric propagation delay [3].

Doppler measurements together with range and accumulated carrier phase are widely used by recursive Kalman filters to accurately determine user's velocity along with their position whether for air, outdoor terrestrial or indoor terrestrial applications [4-5].

For simulation purposes, we have investigated the stationary and moving user scenarios. The error on the Doppler measurement was estimated from recorded Doppler measurements at the Satellite Navigation Lab at WPI.

Summary and conclusions follow the last part of the paper.

Algorithm Description

1. One dimensional case

Assume that one fixed transmitter A and one receiver B are positioned in a one-dimensional axis as shown in figure 1.

We further assume that the receiver B is perfectly synchronized with transmitter A and that the initial location of receiver B is known. Let $d_{AB}^k$ denote the geometric distance between the transmitter A and the receiver B at discrete time $k$. Under the assumption that the receiver B moves toward transmitter A by the amount of $\bar{v}_{AB}^k$, which is analytically determined from:

$$\bar{v}_{AB}^k \approx d_{AB}^{k+1} - d_{AB}^k.$$  (1)

Assuming that the observations are performed at a 1 Hz rate, relationship (1) gives the formula for approximated average velocity during 1 sec interval. We can rewrite (1) assuming that A is fixed,

$$\bar{v}_{AB}^k \approx x_B^{k+1} - x_B^k.$$  (2)

This expression enables us to form the navigation equation for the one-dimensional case in accordance with,

$$x_B^{k+1} \approx x_B^k + \bar{v}_B^k.$$  (3)

It is evident in this case that by knowing the Doppler at any epoch we can determine the location of the receiver B with perfect accuracy in the one-dimensional case.

Note: The same expression can be obtained when the receiver B moves away from the transmitter A with the only modification of (3): the plus sign, (+), now becomes minus, (-).
2. Two dimensional case

At least two transmitters are required to determine the correct location of a single receiver in a two-dimensional plane.

Figure 2: Two-dimensional system diagram

Therefore, we assume that the two transmitters A and B are located in the same plane with the moving receiver C as shown in figure 2. We will again assume that the two transmitters are fixed with the coordinate system under consideration. We will also assume that the initial location of the receiver B is known. Before going on, it can be shown analytically that for A and B fixed and for known distances AC and BC there are two solutions in which C can be located. Both points are symmetrical with respect to the line that passes between A and B. With these assumptions in mind we move now to the derivation of the navigation equations.

At the \((k+1)\)th epoch the receiver C is found at the location as shown in figure 2. It is clear that the new ranges between the receiver C and the transmitters A and B are respectively \(d^{k+1}_{AC}\) and \(d^{k+1}_{BC}\). Similar to the one-dimensional case the receiver C moves toward the transmitters A and B by the amount of,

\[
\bar{v}^{k}_{AC} \equiv d^{k+1}_{AC} - d^{k}_{AC},
\]

\[
\bar{v}^{k}_{BC} \equiv d^{k+1}_{BC} - d^{k}_{BC}.
\]

The new point \(C^{k+1}\) is found from the intersection of two circles—one with its center at the transmitter A and with radius given by (6) and the other circle with its center at the transmitter B and with radius given by (7), as follows,

\[
d^{k+1}_{AC} \equiv d^{k}_{AC} + \bar{v}^{k}_{AC}, \tag{6}
\]

\[
d^{k+1}_{BC} \equiv d^{k}_{BC} + \bar{v}^{k}_{BC}. \tag{7}
\]

Without showing all the work the solution for the new location of the receiver C is determined from:

\[
x^{k+1}_{C} = \frac{1}{2}\left(x_{A} + x_{B} + \frac{d^{k}_{AC} - d^{k}_{BC} + \bar{v}^{k}_{AC} - \bar{v}^{k}_{BC}}{x_{B} - x_{A}}\right), \tag{8}
\]

\[
y^{k+1}_{C} = \pm\sqrt{d^{k}_{AC} + \bar{v}^{k}_{AC} - (x^{k+1}_{C} - x_{A})^{2}}. \tag{9}
\]

Since the real receiver is mounted in a physical object which contains physical mass and inertia; therefore, it cannot be located at two different points at the same time, as it is against any law of classical mechanics. (Note: This may not be true for phenomena approximated by quantum mechanics law or physics; however, the scenario we are dealing with can be accurately described by the classical mechanics dynamics.)

Next in continuation to the argument stated right above, we determine which \(y^{k+1}_{C}\) should be chosen from the two solutions of (9).

According to the dynamics of the problem the new \(y^{k+1}_{C}\) will be inside the circle with radius the sum of the Doppler measurements and with its center the previous location of the receiver, C. Denote with \(r_{cr}\) the radius of the circle inside of which the new location of the receiver is found, which can be written as,

\[
r^{k}_{cr} = \bar{v}^{k}_{AC} + \bar{v}^{k}_{BC}. \tag{10}
\]

Denote the distance between the current position and the new correct position of receiver by \(d^{k+1}_{CC}\), such as,

\[
d^{k+1}_{CC} = \sqrt{(x^{k+1}_{C} - x^{k}_{C})^{2} + (y^{k+1}_{C} - y^{k}_{C})^{2}}. \tag{11}
\]
Similarly, the distance between the current position of the receiver and the mirror image of the new correct position of the receiver, denoted by $d_{CC}^{k+1}$, can be written as,

$$d_{CC}^{k+1} = \sqrt{(x_c^{k+1} - x_c^k)^2 + (y_c^{k+1} - y_c^k)^2}. \quad (12)$$

**Theorem 1:** Relations (10), (11), and (12) fulfill the following inequality:

$$d_{CC}^{k+1} \leq k_{gr} \leq d_{CC}^k. \quad (13)$$

The analytical proof of this theorem is not presented here; however, we would like to emphasize that the criteria (13) serves as the test criteria for choosing the correct $y$ coordinate of the new receiver location.

### 3. Three dimensional case

In a noise and error free environment, the correct location of the receiver D in the three-dimensional case can be determined with help of the three transmitters (A, B, and C), which are not in the same line as shown in figure 3. Let $D^{k+1}$ determine the location of the receiver D at the $(k+1)^{th}$ epoch (see figure 3). It is clear that the new ranges between receiver D and transmitters A, B and C are respectively $d_{AD}^{k+1}$, $d_{BD}^{k+1}$, and $d_{CD}^{k+1}$.

Similar to the one-dimensional and two-dimensional cases, the receiver D moves toward transmitters A, B, and C by the amount of,

$$\tilde{v}_{AD}^k \approx d_{AD}^{k+1} - d_{AD}^k, \quad (14)$$

$$\tilde{v}_{BD}^k \approx d_{BD}^{k+1} - d_{BD}^k, \quad (15)$$

$$\tilde{v}_{CD}^k \approx d_{CD}^{k+1} - d_{CD}^k. \quad (16)$$

The new point $D^{k+1}$ is found from the intersection of three spherical surfaces—one with center at the transmitter A and with radius given by (17), one circle with center at the transmitter B and with radius given by (18), and the final circle with center at the transmitter C and with radius given by (19), as follows,

$$d_{AD}^{k+1} \approx d_{AD}^k + \tilde{v}_{AD}^k, \quad (17)$$

$$d_{BD}^{k+1} \approx d_{BD}^k + \tilde{v}_{BD}^k, \quad (18)$$

$$d_{CD}^{k+1} \approx d_{CD}^k + \tilde{v}_{CD}^k. \quad (19)$$

Without showing all the work the solution for the new location of the receiver D is determined from:

$$x_D^{k+1} = \frac{A_1^k(y_B - y_C) - A_2^k(y_B - y_A)}{(x_B - x_A)(y_B - y_C) + (x_B - x_C)(y_B - y_A)}, \quad (20)$$

$$y_D^{k+1} = \frac{A_1^k(x_A - x_C) - A_2^k(x_B - x_C)}{(x_B - x_A)(y_B - y_C) + (x_B - x_C)(y_B - y_A)}, \quad (21)$$

$$z_D^{k+1} = \pm \sqrt{d_{CD}^{k+1} - \left(x_D^{k+1} - x_C^k\right)^2 - \left(y_D^{k+1} - y_C^k\right)^2}. \quad (22)$$

In expressions (20) and (21) the parameters $A_1^k$ and $A_2^k$ correspond to:

$$A_1^k = d_{AD}^k - d_{BD}^k + \tilde{v}_{AD}^k - \tilde{v}_{BD}^k + x_B^2 - x_A^2 + y_B^2 - y_A^2, \quad (23)$$

$$A_2^k = d_{BD}^k - d_{CD}^k + \tilde{v}_{BD}^k - \tilde{v}_{CD}^k + x_C^2 - x_B^2 + y_C^2 - y_B^2. \quad (24)$$

A criteria similar to theorem 1 for the two-dimensional case can be utilized here to determine the correct $z_D^{k+1}$ for the three dimensional case.
4. Multidimensional noisy (final) case

Consider the case now when more than 4 fixed transmitters are used to determine the location of a single receiver. Suppose that the Doppler measurement vector provided from the receiver at the $k^{th}$ epoch can be written as,

\[ \phi^k = d^k + c(\Delta f^k - \Delta F^k) + e_\phi, \]  

(25)

where,

- $d^k$ is the true range-rate vector,
- $\Delta f^k$ is the receiver clock drift vector,
- $\Delta F^k$ is the transmitter's clock drift vector,
- $e_\phi$ is the Doppler measurement noise vector.

Under the assumption that the location of the receiver at the $k^{th}$ epoch is known, we seek to determine its location at the next epoch. Therefore, instead of the true range between the next receiver location and the fixed transmitters, we will have the Doppler Derived (DD) pseudorange (this is different from the true pseudorange, which is provided from the receiver). The analytical expression of the DD pseudorange vector is given by,

\[ \rho^{k+1} = s^k + \phi^{k+1}. \]  

(26)

Denote the state vector by,

\[ s^k = [x^k, y^k, z^k, \Delta f^k]^T. \]  

(27)

For reasons explained later in the paper, we form the residual vector expressed as,

\[ r^{k+1} = \rho^{k+1} - H^{k+1} \rho^{k+1}. \]  

(28)

We will seek a solution for the new state vector, which minimizes the norm of the residual vector as follows,

\[ \min \| r^{k+1} \| = \min \left( \rho^{k+1} - H^{k+1} s^{k+1} \right)^T (\rho^{k+1} - H^{k+1} s^{k+1}). \]  

(29)

Provided that $H^{k+1\top} H^{k+1}$ is nonsingular that the solution of (29) is determined from,

\[ \hat{s}^{k+1} = \left( H^{k+1\top} H^{k+1} \right)^{-1} H^{k+1\top} \rho^{k+1}. \]  

(30)

Along with solution (30) another valid solution, when the measurement error covariance matrix, $R$, is provided, reads,

\[ \hat{s}^{k+1} = \left( H^{k+1\top} R^{-1} H^{k+1} \right)^{-1} H^{k+1\top} R^{-1} \rho^{k+1}, \]  

(31)

which is based on the criteria $\min \left( s^{k+1} \right)^T R^{-1} \left( s^{k+1} \right)$.

**Note 1:** The advantage of this algorithm compared to the ordinary least square solution, which uses only true pseudoranges, consists in the small diagonal values of the covariance matrix formed with DD pseudorange. This occurs for the reasons explained below. Let $e_\rho^{k+1}$ denote the navigation state error vector. The Doppler measurement noise vector, $e_\phi^{k+1}$, can be translated into the state error vector, $e_\rho^{k+1}$, in accordance with,

\[ e_\rho^{k+1} = \left( H^{k+1\top} \right)^{-1} H^{k+1\top} e_\phi^{k+1}. \]  

(32)

Assuming that for a set of 4 or more pseudolites and for a single receiver we estimate coherently and in parallel the receiver position using the conventional LSA (or CLSA) on the one hand and the modified LSA algorithm (or MLSA) on the other. Without considering the effect of the geometry, there is an improvement of the navigation accuracy utilizing the Doppler MLSA versus the pseudorange CLSA by the ration of $\sigma_\rho / \sigma_\phi$. For typical GPS receivers this number would be greater than 50.

**Note 2:** For the case when only a set of 4 pseudolites is available for navigation then the solution is reduced to:

\[ s^{k+1} = H^{k+1\top} \rho^{k+1}. \]  

(33)

**Note 3:** Quite often a pretty good estimate of the starting point is provided. For this case the challenge is to maintain the same accuracy during
the navigation. We can modify the solution given by (30), (31), and (33) as follows,

\[
\Delta s^{k+1} = \left( \mathbf{H}^{k+1 \ast} \cdot \mathbf{H}^{k+1} \right)^{-1} \cdot \mathbf{H}^{k+1 \ast} \cdot \mathbf{H}^{k+1} \cdot \mathbf{s}^{k+1},
\]

(34)

\[
\Delta \hat{s}^{k+1} = \left( \mathbf{H}^{k+1 \ast} \mathbf{R}^{-1} \mathbf{H}^{k+1} \right)^{-1} \cdot \mathbf{H}^{k+1 \ast} \mathbf{R}^{-1} \cdot \mathbf{H}^{k+1} \cdot \mathbf{r}^{k+1},
\]

(35)

\[
\Delta \hat{s}^{k+1} = \mathbf{H}^{k+1 \ast} \cdot \mathbf{H}^{k+1} \cdot \mathbf{r}^{k+1}.
\]

(36)

Once the solution estimate increment is determined by either (34), (35), or (36) then the absolute solution estimate is easily computed from:

\[
\hat{s}^{k+1} = \hat{s}^{k} + \Delta \hat{s}^{k+1}.
\]

(37)

The benefit of using (34-37) versus (30), (31), and (33) will be demonstrated from the simulation results.

Simulation

Simulation results are provided for one fixed scenario and one moving scenario.

1. Fixed Scenario

This is the scenario in which the moving receiver is assumed stationary. This scenario is important because it is by far the easiest and the simplest one.

![Figure 4: Pseudolite, fixed receiver layout](image)

The pseudolite layout and the receiver layout are shown in figure 4. The receiver is 4 m above the plane of the pseudolites. Considering our interest in the indoor geolocation applications, we have depicted this scenario.

- Raw pseudorange CLSA

First, we process raw pseudoranges using the CLSA algorithm and obtain the lateral and vertical position error for pseudorange measurement error (1 sigma) ranging from 0.01 to 5.12 m (see figures 5 and 6).

- Raw pseudorange MLSA

Next, we repeat the previous test utilizing the MLSA algorithm (see figures 7 and 8).

- DD pseudorange CLSA

![Figure 5: Lateral position error vs. pseudorange measurement noise CLSA](image)

![Figure 6: Vertical position error vs. pseudorange measurement noise CLSA](image)

Next, we process DD pseudoranges employing the CLSA and obtain the lateral and vertical position error for Doppler measurement error (1 sigma) ranging from 0.01 to 0.1 m/sec (see figures 9 and 10).
2. Moving Scenario

The moving scenario depicted in figure 9 was selected from one the indoor geo-location applications [5].

- DD pseudorange MLSA

Next, we repeat the previous test utilizing the MLSA algorithm (see figures 11 and 12).
First we process raw pseudoranges using CLSA and obtain the lateral and vertical position error for pseudorange measurement error (1 sigma) ranging from 0.01 to 5 m (see figures 14 and 15).

Next, we repeat the previous test utilizing the MLSA algorithm (see figures 16 and 17).

Next, we process DD pseudoranges employing the CLSA and obtain the lateral and vertical position...
error for Doppler measurement error (1 sigma) ranging from 0.01 to 0.1 m/sec (see figures 18 and 19).

- DD pseudorange MLSA

Next, we repeat the previous test utilizing the MLSA algorithm (see figures 20 and 21).

For distances comparable to 1 sigma value, the CLSA algorithm provides a better accuracy than the MLSA algorithm.

References