GPS AND PSEUDO-SATELLITES INTEGRATION FOR PRECISE POSITIONING

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ABSTRACT

It is well known that for spaceborne satellite positioning systems such as GPS and Glonass, the accuracy, availability and reliability of the positioning results is heavily dependent on the number and geometric distribution of satellites being tracked. However, in some situations, such as in urban canyons and in deep open-cut mines, the number and geometry of visible satellites may not be sufficient to reliably carry out positioning operations. These problems can be addressed by the inclusion of additional ranging signals transmitted from ground-based "pseudo-satellites" (pseudolites). In this paper the authors present details of both theoretical and experimental investigations into the potential integration of GPS and pseudolite technologies for precise positioning applications. The experiments indicate that, with integrated GPS and pseudolite signals, the accuracy and reliability of position results can be improved compared to GPS-only results.
INTRODUCTION

Applications of artificial satellites in navigation and positioning can be traced back to as early as the 1960’s, just shortly after the launch of the first artificial satellite in 1957. Over the past four decades, particularly since the inception of the Global Positioning System (GPS), satellite-based positioning techniques have been playing an increasingly important role in surveying and geodesy. In comparison with the traditional surveying techniques, one of the advantages of satellite-based positioning systems is that there is no requirement for line-of-sight between two survey marks (control points), offering significantly greater flexibility for positioning operations. In such satellite-based systems, range measurements are derived using the direct satellite ranging signals received by the receiver antennas mounted over survey marks. This, however, requires that the line-of-sight between the satellites and survey marks be maintained during the measurement process, which unfortunately pose some limitations to satellite-based positioning systems.

In satellite-based precise positioning, the dominant factors are the number and geometric distribution of the satellites tracked by the receivers. In the case of global navigation satellite systems such as GPS, Glonass, and the planned Galileo system, four visible satellites are the minimum requirement for precise three-dimensional positioning. In general, the more satellites that are tracked, the more reliable the positioning solutions. However, in some situations, such as in downtown urban canyons and in deep open-cut pits and mines, the number of visible satellites may not be sufficient.

Furthermore, the geometric distribution of the satellites being tracked will have a significant impact on the accuracy of the estimated baseline components. It is well known that the horizontal components of baseline solutions are much better determined than the height component. The reason for this is that very low elevation satellites are not tracked. Even when some low elevation satellites can be tracked, the measurements from these satellites are not of high quality.

The above problems with spaceborne satellite positioning systems can be addressed by the use of additional ranging signals transmitted from ground-based pseudo-satellites, generally referred to as pseudolites (PLs). In fact, in 1970’s, even before the launch of the GPS satellites, pseudolites had been used to test the initial GPS user equipment (Harrington & Dolloff, 1976). In the mid 1980s, the RTCM committee SC-104 ('Recommended Standards for Differential Navstar GPS Service') designated the Type 8 Message for the pseudolite almanac, containing the location, code and health information of pseudolites (Kalafus et al., 1986). Investigations into pseudolites for aviation, marine and land navigation, and other positioning applications have intensified recently (for example, Bartone, 1996; Choi et al., 2000; Cobb et al., 1995; Ford et al., 1996; Hein et al., 1997; Morley, 1997; Stone & Powell, 1998; Zimmerman & Cannon, 1996).

In May 1999, the Satellite Navigation And Positioning (SNAP) group in the School of
Geomatic Engineering, of The University of New South Wales (UNSW), purchased the first pseudolite in Australia – the IntegriNautics IN200CXL instrument. Investigations have been conducted in order to explore the applications of pseudolites in precise positioning and navigation. The goal is to develop a suitable GPS augmentation system to raise the precise positioning 'productivity' for those applications which require uninterrupted GPS-based positioning.

In this paper, some modelling issues concerning pseudolite applications are discussed, and experimental results are presented to demonstrate capabilities of an integrated GPS and pseudolite positioning system.

MODELLING PSEUDOLITE MEASUREMENTS

From the positioning point of view, a pseudolite can be considered to be a 'satellite-on-the ground' because the pseudolites transmit GPS-like signals. Therefore, the fundamental principles in precise GPS positioning, such as interferometry, can be applied to integrated GPS-pseudolite positioning.

Measurement models

In principle, pseudolites can transmit their ranging signals on different frequencies, just as the Glonass satellites do. In fact Glonass-style pseudolites are predicted to have a better performance in overcoming the near/far ratio problem (Galijan & Lucha, 1993). Currently, the selection of the optimal frequencies for pseudolites apart from the GPS frequencies is still under investigation. Without loss of generality, in a manner similar to the GPS/Glonass satellite measurement equations (for example, Wang 2001; Wang et al. 2001), the mathematical model for the pseudolite pseudo-ranges and carrier phases are:

\[ R^p_k = \rho^p_k + c \cdot (dt^p - dt_k) + T^p_k + dr^p_k + dm^p_k + \epsilon^p_k, \]  
\[ \phi^p_k = \frac{1}{\lambda^p} \rho^p_k + \frac{c}{\lambda^p} \cdot (dt^p - dt_k) + N^p_k + \frac{1}{\lambda^p} T^p_k + \frac{1}{\lambda^p} dr^p_k + \delta m^p_k + \epsilon^p_k \]

where \( R^p_k \) and \( \phi^p_k \) are pseudo-range and carrier phase measurements from receiver \( k \) to pseudolite \( p \) respectively; \( \lambda^p \) is the wavelength of the carrier frequency for pseudolite \( p \); \( \rho^p_k \) is the topocentric distance between receiver \( k \) and pseudolite \( p \); \( c \) is the speed of light; \( dt^p \) is the pseudolite clock error; \( dt_k \) is the receiver clock error; \( N^p_k \) is the integer carrier phase ambiguity; \( T^p_k \) is the tropospheric delay; \( dr^p_k \) is the pseudolite location error; \( dm^p_k \) and \( \delta m^p_k \) are multipath errors in the pseudo-range and carrier phase.
respectively; and $\varepsilon_k^p$ and $\epsilon_k^p$ are pseudo-range and carrier phase measurement errors respectively.

From the above equations (1) and (2) it is noted that there is no signal-propagation correction terms for the ionosphere. This is because pseudolite signal transmitters and the user receiver antennas are both ground-based. Hence the pseudolite signals will not propagate through the ionosphere, which lies approximately between 50km and 1000km above the surface of the Earth. However, unlike the spaceborne satellite signals, the pseudolite signals propagate through the lower troposphere. It is due to this fact that the effects of tropospheric delays in the pseudolite measurements should be corrected using a special tropospheric delay compensation model or are estimated using pseudolite measurements (Barltrop et al., 1996; Morley, 1997; Hein et al. 1997; Dai et al., 2000). The tropospheric correction to pseudolite measurements is similar to that for electronic distance measurement equipment. A detailed discussion on this topic can be found in Rüeger (1996).

In precise satellite and pseudo-satellite positioning, the double-differencing procedure is applied to cancel out or eliminate the (pseudo-)satellite clock errors and receiver clock errors. When pseudolites transmit their ranging signals on frequencies which are different from GPS L1/L2 frequencies, the integration of GPS and pseudolites is similar to the situation of GPS and Glonass integration. Therefore, the modelling strategies developed for integrating GPS and Glonass measurements can be applied when combining pseudolites with GPS (or other Global Navigation Satellite Systems such as Glonass and Galileo). If pseudolites, such as the IntegriNautics IN200CXL used in this research, transmit their ranging signals on the GPS L1 (and/or L2), integrating the GPS and pseudolite measurements is relatively straightforward, in the sense that pseudolites are simply extra GPS satellites 'on the ground'.

Because of the low elevation angle of the pseudolite signals, the multipath error on the pseudo-range and carrier phase measurements are usually larger than is the case for GPS measurements. Unlike the spaceborne satellites, pseudolites are static during the process of positioning, hence it is expected that the effects of multipath, and the pseudolite location errors, will be a ‘constant’ in the case of static positioning applications. This issue will be discussed later, together with the experimental results.

The use of ground-based pseudolites will have other implications. For example, for short baselines, the effects of nonlinearity on the measurement equations are negligible for the satellite transmitters, but could be significant in the case of pseudolites located on the ground. The reason for this is that pseudolites are much closer to user receivers than the satellites are.

**Effects of Nonlinearity**

In positioning applications, the key geometric information from the measurements is distance or range between two points, which are generally represented through coordinates defined in some reference frame. The relationship between the measurements (distances) and the unknown parameters (coordinates) is of course
nonlinear. Because the estimation techniques for the linear models have attractive statistical properties, the nonlinear measurement equations, such as (1) and (2), are usually linearized using a Taylor series expansion.

A function $F(x)$ is expanded into a Taylor series up to the first order as:

$$F(x) = F(x_0) + A\delta x + R$$

where $x_0$ is the approximate values for the parameters $x$; $A = \partial_x F(x_0)$ is the vector of first order partial derivatives evaluated at $x_0$, which is also called the line-of-sight (LOS) vector; $\delta x = x - x_0$; and $R$ is the second order remainder term:

$$R = \frac{1}{2} \delta x^T \partial_{xx}^2 F(x_0 + t \cdot \delta x) \delta x, \quad 0 < t < 1$$

where $\partial_{xx}^2 F(x)$ is the Hessian matrix constructed using the second order partial derivatives. In standard data processing, the remainder term $R$ is ignored, resulting in a nonlinearity error in the measurement model. The bounds of this error term are given by (Teunissen, 1987):

$$\frac{1}{2} \lambda_{\text{min}} \|\delta x\|^2 \leq R \leq \frac{1}{2} \lambda_{\text{max}} \|\delta x\|^2$$

where $\lambda_{\text{min}}$ and $\lambda_{\text{max}}$ are the minimum and maximum eigenvalues of the Hessian matrix $\partial_{xx}^2 F(x)$. For the distance model:

$$d_{ij} = \sqrt{x_{ij}^2 + y_{ij}^2 + z_{ij}^2},$$

where $x_{ij}$, $y_{ij}$, $z_{ij}$ are the differences between the coordinates for the two points $(i, j)$ associated with the measurement, the Hessian matrix is:

$$\partial_{xx}^2 d_{ij} = \frac{1}{d_{ij}^3} \begin{bmatrix} y_{ij}^2 + z_{ij}^2 & -x_{ij} y_{ij} & -x_{ij} z_{ij} \\ -x_{ij} y_{ij} & x_{ij}^2 + z_{ij}^2 & -y_{ij} z_{ij} \\ -x_{ij} z_{ij} & -y_{ij} z_{ij} & x_{ij}^2 + y_{ij}^2 \end{bmatrix}$$

with the extreme eigenvalues being $\lambda_{\text{min}} = 0$ and $\lambda_{\text{max}} = 1/d_{ij}$. From equation (5), the bounds for the nonlinearity error caused by ignoring the remainder term $R$ are:
In integrated GPS-pseudolite positioning, the distances for linearization may vary from 20000km between GPS satellites and the user, to 200m (or even as short as a few metres) in the case of pseudolite(s)-user separation. Figures 1 and 2 show the maximum linearization errors (bounds) for these two different separations.

For typical distances between GPS satellites and users on the ground, the linearization error for a 200m error in one component of the coordinates is just 1mm. However, when the separation between pseudolites and users is just 200m, an error of 15m in one component of the coordinates may result in a linearization error as much as 0.6m, which is much larger than the phase measurement errors and may lead to divergence of the computation process. To prevent this from happening, at the beginning of the least-
squares computation, a large variance for the pseudolite measurements is introduced (Elrod & Van Dierendonck, 1996). In most cases, if multipath effects are not significant, differential pseudo-range solutions will provide initial coordinates of sufficient accuracy for iterative processing of nonlinear measurements.

**EXPERIMENTAL RESULTS**

A variety of static experiments have been conducted at UNSW to investigate the potential application of pseudolites for precise positioning. The IntegriNautics IN200CXL pseudolites and NovAtel Millennium GPS receivers were used in the experiments. In all the experiments the pseudolite signals (GPS L1) were pulsed with RTCM mode at a 1/11 duty cycle. The PRN number for the pseudolite signals was configured as 32. The data sampling rate was set to 1Hz.

The standard double-differencing (DD) procedure was used to process the combined GPS and pseudolite measurements. The existing GPS/Glonass data processing software was modified to accommodate the pseudolite measurements. The following data analysis focuses on the noise levels and systematic error characteristics of pseudolite pseudo-range and carrier phase measurements, and the contributions of pseudolite signals to precise positioning.

**Noise Levels**

A zero-baseline test was conducted to compare the GPS and pseudolite signal noise levels. For a zero-baseline, the DD residuals mainly reflect the receiver tracking performance as the orbital errors, atmospheric errors, multipath and antenna-related systematic errors are removed. The pseudolite was set up about 150m away from the GPS receiver antenna. In the data processing, the highest satellite was selected as the reference satellite. Figures 3 and 4 show the DD carrier phase residuals from the zero-baseline solution. The UTC time of the first epoch data was 0:20:00.0 of 14th March 2000.

![Fig. 3: DD carrier phase residuals for SV3 - SV23 (zero-baseline).](image_url)
It should be noted that the pseudolite carrier phase noise level (with a RMS of 0.007cycles) is higher than the GPS carrier phase noise level (with a RMS of 0.005cycles). For the pseudo-range measurements, similar trends have also been identified. In the case of GPS, the noise levels have a relationship to the elevation angle of the satellites (see further discussion below). Whilst the average elevation for SV23 was about 17 degrees, the pseudolite (PL32) had a constant elevation of about 7 degrees. It could be construed that the GPS receivers might have a similar performance in the case of combined GPS/pseudolite signal tracking.

**Systematic Errors**

In real applications, however, the unmodelled systematic errors will become the dominant error sources. To investigate these, a static experiment was conducted on the lower portion of roof of a building on the UNSW campus (see Figure 5).

![Fig. 4: DD carrier phase residuals for SV3 - PL32 (zero-baseline).](image)

![Fig. 5: Set-up of static GPS/PL baseline determination experiment.](image)
In this experiment the baseline length was approximately 3m. The separation between the pseudolite and the baseline was about 10m. There are 20 concrete pillars and several metal structures on this portion of the roof, hence the observing site is considered to be subject to multipath reflections. The elevation angle of the pseudolite was about 6 degrees, whilst the elevation angles for all five tracked satellites were greater than 40 degrees. The UTC time of the first epoch data was 11:48:00.0 of 16th April 2000.

Figures 6 and 7 show the DD carrier phase residuals from the ambiguity-fixed baseline solutions (the highest satellite SV2 was used as the reference satellite). These residuals indicate that the measurements were contaminated by systematic errors, which are most likely due to multipath. Naturally, in the case of GPS satellites as shown in Fig. 6, due to the changing satellite geometry, the remaining systematic errors in the satellite measurements vary with time (with a low frequency).

Fig. 6: DD carrier phase residuals for SV2 – SV7 (3m baseline).

Fig. 7: DD carrier phase residuals for SV2 – PL32 (3m baseline).
In the case of pseudolites, there is no geometry change. The systematic errors caused by, for instance, multipath and pseudolite-location errors, will exhibit a constant characteristic in pseudolite measurements. This phenomenon is shown in Fig. 7. Such constant biases in pseudolite measurements have also been mentioned in other investigations (for example, Choi et al., 2000; Dai et al., 2000; Ford et al., 1996; Hein et al., 1997). The evaluation of such a constant bias need not be just based on the residuals of the ambiguity-fixed solutions. The reason is that the pseudolite measurements will have a significant influence in the solutions, particularly for the height components. The bias in the pseudolite measurements may be ‘absorbed’ by the GPS measurements.

Assuming the pseudolite measurements are expected to fit the GPS measurements, the constant bias in the pseudolite measurements could be estimated and removed using a search algorithm such as proposed by Wang (2000, 2001). The idea behind this algorithm is that all the possible bias values are tested to identify the most likely one, which produces the lowest RMS for the residuals. For this application of the algorithm, the search step could be selected as 0.005 cycles (noise level of the carrier phase measurements). For this 3m baseline, the constant bias was estimated to be 0.115 cycles.

In contrast to the zero-baseline test, in this short baseline test the noise level (which might contain some of the ‘randomized’ systematic errors) of the pseudolite carrier phase measurements is much lower than that of the GPS carrier phase measurements. On the other hand, the noise level of the pseudolite measurements from this short baseline is even better than that from the zero-baseline test, although in both experiments the pseudolite elevation angles are almost identical and the receivers are the same. The different noise levels of the measurements might be caused by varying pseudolite signal strength. It is noted that, in the RINEX files, the pseudolite signal strengths were scaled at 5 and 9, for the zero and 3m baselines respectively.

These experiments demonstrate that the quality of the pseudolite carrier phase measurements could be at the same level as, or even better than, that of the GPS measurements. However, the challenging issue is the efficient mitigation of the biases in both the carrier phase and pseudo-range measurements, which may reach several metres or more (Choi et al., 2000; Hein et al., 1997). Once the biases are negligible or properly accounted for, the valuable contribution of pseudolite measurements in the positioning process becomes clearer.

**Positioning Results**

A static GPS-pseudolite positioning experiment was conducted on the lower roof of a UNSW building (see Figure 5). Antenna 1 was selected as the reference station. The baseline length between antenna 1 and antenna 2 is about 3 metres. As all the antennas are stationary, the performance of the integrated GPS-pseudolite positioning can be evaluated.

During the experiment on 16 April 2000, four satellites were tracked. The elevation angles to these satellites were above 30 degrees (30°, 48°, 55°, 60°). The elevations of
the pseudolite transmitter at these two antennas were about 5.0 and 6.0 degrees respectively.

As the first step in the data analysis (single-epoch solutions), the GPS-only solution was obtained. For the integrated GPS and pseudolite positioning, as discussed above, pseudolite carrier phase biases could be identified. Several solutions were obtained, based on the following treatment of the biases:

A) Biases are ignored.
B) Biases are estimated using the whole data set, and then are used to correct the pseudolite phases for every epoch.
C) Biases are estimated using the measurements in one epoch, and then used to correct the pseudolite phases in the current epoch.

The statistics regarding both three-dimensional baseline components are listed in Table 1. Figures 8, 9 and 10 show the height component values from the four different solutions.

<table>
<thead>
<tr>
<th>Solution Types</th>
<th>North (m)</th>
<th>East (m)</th>
<th>Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std Dev</td>
<td>Mean</td>
</tr>
<tr>
<td>GPS-only</td>
<td>3.243</td>
<td>0.0053</td>
<td>1.033</td>
</tr>
<tr>
<td>GPS/PL A</td>
<td>3.244</td>
<td>0.0058</td>
<td>1.028</td>
</tr>
<tr>
<td>GPS/PL B</td>
<td>3.243</td>
<td>0.0052</td>
<td>1.033</td>
</tr>
<tr>
<td>GPS/PL C</td>
<td>3.243</td>
<td>0.0056</td>
<td>1.033</td>
</tr>
</tbody>
</table>

Fig. 8: Height component values from the GPS-only solutions.
From these Figures and the Table, the contribution of the pseudolite measurements to the baseline solutions is clearly significant for the height component (solution A & B). It is interesting to note from Fig. 9 that, even without calibrating the biases in the PL carrier phases, the height component parameter could be precisely estimated, although a shift of 0.02m from GPS-only solution exists. This shift could be removed if the bias in the PL measurements is estimated reliably, and then corrected for. However, GPS/PL solution C indicates that the reliable estimation of the bias requires strong geometric strength, and a single epoch of data is not sufficient for this purpose.
CONCLUDING REMARKS

The reliability and accuracy of satellite-based positioning are highly dependent on both the number of visible satellites and their geometric distribution. The integration of pseudolite and GPS signals is one of the options for improving system performance, particularly in poor operational environments. The potential of such an option has been demonstrated in this paper.

The effects of nonlinearity have been analysed from a theoretical point of view. The formulas derived in this paper show that special attention has to be given to these error sources in pseudolite positioning applications. The experimental results have shown that the noise level of the PL carrier phase measurements is comparable with, or even lower than, that of GPS measurements. The baseline solutions can be significantly improved by adding PL measurements with low elevation angles. However, the mitigation of the (unmodelled) systematic errors identified in the PL measurements is a challenging issue for further investigation.

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REFERENCES


