Adaptive Spatial and Temporal Selective Attenuator in the Presence of Mutual Coupling and Channel Errors

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Biography
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Abstract
An adaptive spatial and temporal selective attenuator (ASTSA) can be used to mitigate the effects of undesired narrow and/or wideband signals. This can be accomplished by applying a set of multipliers, which seeks to maximize the sensitivity in the direction of the desired signal(s) and minimize the sensitivity in the direction of the undesired signal(s). This minimization and/or maximization criterion is defined as the undesired over desired ratio (UDR). The presence of mutual coupling and channel errors will distort both the desired and the undesired signal(s) thus producing a different set of multipliers utilizing the UDR ratio. The effect of mutual coupling and channel errors on the UDR ratio is analyzed and presented in this paper.

Introduction
The quest to mitigate the undesirable effects of the narrow and/or wide band signals starts with works by Friis and Feldman [1] and by Price [2]. Although this research proposes ideas such as utilizing a set of complex weights to mitigate the effect of undesired signals at specified directions, these ideas were impractical when they were presented due to the existing level of technology.

In the early 1970's the rapid development of digital electronics and the early development of minicomputers enabled faster processing, which led to novel applications and utilization of the ideas once considered impractical. A classical work by Applebaum [3] suggests an adaptive technique for optimizing the signal-to-noise ratio, thus reducing the array susceptibility to jamming or interference of any kind. Applebaum’s technique, however, leaves out the consideration of the desired signal, which is mandatory for communication devices in which the desired signal is continuously present (introduced by Frost) [4].

The advances in technology facilitated size reduction of the adaptive arrays and improved signal processing performance. Size reduction of the adaptive array was achieved by bringing the array elements closer, which led to noticeable effects due to the mutual coupling between array elements. Techniques, such as one proposed by Steysal and Herd, were then developed which attempt to restore the signal from the deteriorating effect of mutual coupling [5]. Despite the sound analytical derivation and experimental observations of this technique, its implementation requires constant calibration of the array elements, which increases maintenance cost. The effects of these unavoidable array errors in the adaptive beamforming was analyzed and simulated against the Applebaum criteria [3] and the Frost criteria [4], by Stayskal [6]. The end result of this work seems to indicate that the small array errors leave an unchanged beam pattern when the Applebaum criteria is used [4]; the same small errors, however, produce a surprisingly different beam pattern when the Frost criteria is used, although...
maintaining the same null depth in the direction of the interference.

Later advances in digital beamforming techniques [7,8] appear to indicate that the desired antenna array pattern and the desired null depth in the direction of the interference remain unaffected by the presence of both frequency dependent mutual coupling and channel errors utilizing either an open or closed adaptive beamforming loop.

In this paper, we revisit the effect of both mutual coupling and channel errors by considering an adaptive array with temporal degrees of freedom as well as spatial degrees of freedom. The resulting array attempts to adaptively and selectively attenuate a combination of undesired interference sources by employing a generalized Frost criterion. A series of analysis and simulation presents the ASTSA performance in the presence of both mutual coupling and channel errors.

**Ideal ASTSA (Without Mutual Coupling or Channel Errors)**

Consider an adaptive temporal and spatial selective attenuator as pictured in Figure 1.

![Figure 1: The adaptive spatial and temporal selective attenuator block diagram](image)

Denote the desired signal at the operational frequency, OF, as seen by the $k$th antenna array element, $\forall k \in \{1, \cdots, A\}$, by,

$$d_k(t) = D(t, W) \exp\left[\left(\omega_{D} t + \alpha_{d} + \beta_d\right)\right].$$  \(1\)

The undefined terms in equation (1) are defined in order, $W$ denotes the signal bandwidth, $D(t, W)$ denotes the modulated/un-modulated signal amplitude, $\omega_{D}$ denotes the operational signal radian frequency, $\alpha_{d}$ is the phase shift of the signal as seen by the $k$th element array with respect to the reference element array, $\beta_d$ is the initial phase of the desired signal, $t$ of course denotes the time duration.

In our notation convention, the subscript $d$ defines the desired components of the signal; the subscript $u$ defined the undesired components of the signal; and subscript $o$ is assigned to operational frequency components of the signal.

Similarly, denote the undesired signal at the operational frequency, OF, as seen by the $k$th antenna array element, $\forall k \in \{1, \cdots, A\}$, by,

$$u_k(t) = U(t, W) \exp\left[\left(\omega_{U} t + \alpha_{u} + \beta_u\right)\right].$$  \(2\)

The only undefined term in equation (2) is the modulated/un-modulated amplitude of the undesired signal $U(t, W)$.

The total noiseless signal at the operational frequency, OF, as seen by the $k$th antenna array element, $\forall k \in \{1, \cdots, A\}$, is assigned by $s_k(t)$, which is the sum of both signals given by expression (1) and (2),

$$s_k(t) = d_k(t) + u_k(t).$$  \(3\)

We intend to keep the problem simple by considering just two signal sources, one desired and one undesired interference signal. Even for this scenario, the analysis is not straightforward, which justifies this constraint.

The signal, $s_k(t)$, is corrupted by noise due to its transmission through a non-uniform environment, the effect of which is approximated as white noise, $v_k(t)$. Therefore, the total noisy signal at the operational frequency as seen by the $k$th antenna array element, $\forall k \in \{1, \cdots, A\}$, is given by $r_k(t)$,

$$r_k(t) = s_k(t) + v_k(t) = d_k(t) + u_k(t) + v_k(t).$$  \(4\)

For devices which utilize the desired signal, $d_k(t)$, extracting it from the total signal, $r_k(t)$, requires some special efforts. One such effort employs the UDR criterion; it seeks to achieve maximum sensitivity in the direction of the desired signal at the expense of achieving minimum sensitivity in the direction of the undesired signal,

$$UDR = \frac{\min\left(P_o\right)}{\max\left(P_d\right)}.$$  \(5\)

Solving the nonlinear optimization criterion (5) requires extra degrees of freedom, which are obtained in two ways:

1. Forming an antenna element array which uses the phase difference information as observed by the same signal from two antenna array elements to obtain a desired set of multipliers, which are further combined with the total signal.
2. Adding temporal shifters associated with a single antenna array element. Each shifter delays the total signal; therefore, the same phase shift information can be used to obtain a desired set of multipliers, which are further combined with the total signal.

Denote the time delay of the temporal shifter by \( \tau \). The analytical expression of the total noisy signal \( r_k(t) \) coming out of the \( l \)th temporal shifter, \( \forall k \in \{1, \cdots, A\} \), \( \forall l \in \{1, \cdots, B\} \), and \( \vec{t} = t - l \tau \), is given by,

\[
r_k(\vec{t}) = s_k(\vec{t}) + v_k(\vec{t}) = d_k(\vec{t}) + u_k(\vec{t}) + v_k(\vec{t}). \tag{6}
\]

The signal vector for the total signal can be formed as

\[
r(t) = [r_1(t) \cdots r_A(t - B \tau)]^T. \tag{7}
\]

We will denote the combined set of multipliers associated with the antenna array elements and the set of multipliers associated with the temporal shifters by \( \mathbf{m}(t) \):

\[
\mathbf{m}(t) = [m_1^1 \cdots m_B^B \cdots m_A^1 \cdots m_B^B]^T. \tag{8}
\]

We observe that the same signal as seen by two antenna elements or two temporal shifters is highly correlated. Denote with \( C(\delta) \) the correlation matrix defined as,

\[
C(\delta) = E[r(t) \otimes r(t - \delta)^H]. \tag{9}
\]

As defined by expression (9) the correlation matrix \( C(\delta) \) manifests two properties:

1. It is Hermitian symmetric denoted by superscript \( H \);
2. It is positive definite denoted by \( \mathbf{x}^H \cdot C(\delta) \cdot \mathbf{x} > 0 \).

While analyzing this problem, we are particularly interested in the condition when the system is in steady state, which is characterized by \( C(\delta = 0) \).

Finally, we establish the adaptive computation criteria for computing the desired set of multipliers, \( \mathbf{m}(t) \), which satisfy (5),

\[
C(\delta = 0) \cdot \mathbf{m}(t) = \lambda \mathbf{D}. \tag{10}
\]

In equation (10) the constant coefficient \( \lambda \) is determined while normalizing the set of multipliers \( \mathbf{m}(t) \) and \( \mathbf{D} \), the vector pointing in the direction of the desired signal.

It remains now to quantify the UDR expression to relate that with the steady-state correlation matrix \( C(\delta = 0) \), the set of multipliers, \( \mathbf{m}(t) \), and the pointing vector, \( \mathbf{D} \).

Therefore, the denominator of the UDR is the sensitivity gain in the direction of the desired signal and the numerator of UDR is the total output sensitivity. Since the total output sensitivity is an observable quantity; hence, it justifies the purpose of using it as the numerator of UDR.

\[
\text{NUM}(UDR) = \mathbf{m}^H(t) \cdot C(0) \cdot \mathbf{m}(t) \quad \text{and} \quad \text{DEN}(UDR) = P_d [\mathbf{m}^H(t) \cdot \mathbf{D}] [\mathbf{m}^H(t) \cdot \mathbf{D}]. \tag{11}
\]

where \( P_d \) is the sensitivity of the desired signal in the entrance of the antenna element; and thus the UDR ratio reads,

\[
UDR = \frac{\text{NUM}(UDR)}{\text{DEN}(UDR)} = \frac{1}{P_d [\mathbf{m}^H(t) \cdot \mathbf{D}]} \tag{12}
\]

The algorithm for obtaining the desired set of multipliers and analyzing the performance of the adaptive temporal and spatial selective attenuator can be formulated as follows:

1. Compute a simplified expression for the steady-state correlation matrix, \( C(\delta = 0) \), given by (9);
2. Find an expression for the desired pointing direction;
3. Find a simplified expression for the desired set of multipliers given by (10);
4. Estimate UDR according to (12).

We start by computing the diagonal elements of the correlation matrix, \( C(\delta = 0) \), for \( i = f(k,l) \),

\[
c_ii(0) = E[s_k(t - l \tau) \cdot r_k(t - l \tau)^*]. \tag{13}
\]

According to the result obtained for the total noisy signal (6) and assuming that the signal is uncorrelated with the unity variance noise, for \( i = f(k,l) \), yields,

\[
c_ii(0) = s_k(t - l \tau)^2 + 1 + j 0 > 0. \tag{14}
\]

We observe that the diagonal element is a positive real quantity. The off-diagonal elements are obtained from \( (i = f(k,l) \quad \text{&} \quad j = f(n,m)) \):

\[
c_ij(0) = E[s_k(t - l \tau) \cdot r_n(t - m \tau)^*]. \tag{15}
\]

Expanding (15) by assuming zero mean noise \( (i = f(k,l) \quad \text{&} \quad j = f(n,m)) \), produces,

\[
c_ij(0) = E[s_k(t - l \tau) \cdot s_n(t - m \tau)^*]. \tag{16}
\]

Both expression (14) and (16) will be further analyzed later in this paper when some specific scenarios are presented.
In cases where the sensitivity to the desired signal is below the noise power and the sensitivity to the undesired signal is above the noise floor, the problem of selecting coefficients is especially important.

To facilitate signal processing, the signal frequency is downshifted to a transient frequency, TF. The expression for the desired signal at the transient frequency as seen by the \( k \)th antenna array element, \( \forall k \in \{1, \ldots, A\} \), is,

\[
d_k(t) = D(t, W) \exp\left[j(\omega_T t + \alpha_k^d + \beta_d)\right].
\]

The desired signal coming out of the \( m \)th temporal shifter, \( \forall m \in \{1, \ldots, A\} \) & \( \tau = t - m \tau \), is,

\[
d_m(t) = D(\tau, W) \exp\left[j(\omega_T \tau + \alpha_k^d + \beta_d)\right].
\]

The desired signal vector can be formed as,

\[
d(t) = D(t, W)\exp\left[j(\omega_T t + \beta_d)\right] \bar{D},
\]

where,

\[
\bar{D} = \left[ \exp\left[j(\alpha_1^d)\right] \cdots \exp\left[j(\alpha_A^d)\right] \right].
\]

The open form of the desired pointing vector, \( \mathbf{D} \), is presented below:

\[
\mathbf{D} = \left[ \exp\left[j(\alpha_1^d)\right] \cdots \exp\left[j(\alpha_A^d)\right] \right].
\]

For illustration purposes we are considering an array with two antenna elements and one temporal shifter. The steady-state correlation matrix can be presented as \( (\tilde{\tau} = t - \tau, \ s = s(t), \ \tilde{s} = s(\tilde{t})) \):

\[
\mathbf{C} = \begin{bmatrix}
\bar{s}_1^2 & \bar{s}_1 \bar{s}_2 & \bar{s}_2^2 \\
\bar{s}_1 \bar{s}_2 & \bar{s}_1^2 + 1 & \bar{s}_1 \bar{s}_2 \\
\bar{s}_2 \bar{s}_1 & \bar{s}_2 \bar{s}_1 & \bar{s}_2^2 + 1
\end{bmatrix}.
\]

For the correlation matrix given by (22) we will first derive an analytical expression for its determinant. It can be shown that its determinant is equal to \( |\mathbf{C}| = \bar{s}_1^2 + \bar{s}_2^2 + \bar{s}_1 \bar{s}_2 + \bar{s}_2 \bar{s}_1 + 1 \).

The inverse of the correlation matrix can be obtained from:

\[
\mathbf{C}^{-1} = \frac{-1}{|\mathbf{C}|} \begin{bmatrix}
-c_{11} & -c_{12} & -c_{13} & -c_{14} \\
-c_{21} & c_{22} & -c_{23} & -c_{24} \\
-c_{31} & -c_{32} & c_{33} & -c_{34} \\
-c_{41} & -c_{42} & -c_{43} & c_{44}
\end{bmatrix},
\]

where,

\[
c_{11}^{-1} = |\mathbf{C}| - \bar{s}_1^{-2}, \quad c_{22}^{-1} = |\mathbf{C}| - \bar{s}_2^{-2}, \quad c_{33}^{-1} = |\mathbf{C}| - \bar{s}_1 \bar{s}_2, \quad c_{44}^{-1} = |\mathbf{C}| - \bar{s}_1 \bar{s}_2.
\]

It can be easily verified that, \( \mathbf{C} \cdot \mathbf{C}^{-1} = \mathbf{I} \) or \( \mathbf{C}^{-1} \cdot \mathbf{C} = \mathbf{I} \), where \( \mathbf{I} \) is the identity matrix.

The desired set of multipliers which results from (10) is not shown here; nevertheless, before computing the UDR ratio, we calculate the inner product of \( \mathbf{m}^H(t) \cdot \mathbf{D} \) as

\[
\mathbf{m}^H(t) \cdot \mathbf{D} = |\mathbf{C}|^{-1} (3|\mathbf{C}| + 1 - 2 \Re \left[ s_1 \bar{s}_1 + (s_1 s_2^* + s_1 \bar{s}_2^*) \exp\left[j(\alpha_1^d - \alpha_2^d)\right] + (s_1 s_2^* + s_1 \bar{s}_2^*) \exp\left[j(\alpha_1^d - \alpha_2^d)\right] + s_1 \bar{s}_2^* \right]).
\]

At this point we are ready to obtain some numerical results for the following scenario:

**Scenario 1: Nulling Narrowband Interference**

Assume that the desired signal is wideband (20 MHz bandwidth and 30 dB below the noise floor), given by \( d_k(t) = 10^{-15} \sin(20\pi10^4 t)\exp\left[j2\cdot10^4 t + \alpha_k^d\right] \) and that the undesired signal is narrowband (80 dB above the noise floor and 5-deg elevation) determined from \( u_k(t) = 10^4 \exp\left[j2\cdot10^4 t + \alpha_k^u\right] \). What are the minimum and the maximum values of UDR ratio?

\[
s_1 \bar{s}_1^* = A^2 \exp\left[j(\alpha_1^d + \alpha_2^d + \omega_T \tau)\right] + \\
F \exp\left[j(\alpha_1^d - \alpha_2^d + \omega_T \tau)\right] + \\
G \exp\left[j(\alpha_1^d - \alpha_2^d + \omega_T \tau)\right],
\]

\[
s_1 s_2^* = B^2 \exp\left[j(\alpha_1^d - \alpha_2^d)\right] + U^2 \exp\left[j(\alpha_1^d - \alpha_2^d)\right] + F \exp\left[j(\alpha_1^d - \alpha_2^d)\right] + \exp\left[j(\alpha_1^u - \alpha_2^u)\right],
\]

\[
s_1 \bar{s}_2^* = A^2 \exp\left[j(\alpha_1^d - \alpha_2^d + \omega_T \tau)\right] + \\
F \exp\left[j(\alpha_1^d - \alpha_2^d + \omega_T \tau)\right] + \\
F \exp\left[j(\alpha_1^d - \alpha_2^d + \omega_T \tau)\right] + \\
U^2 \exp\left[j(\alpha_1^u - \alpha_2^u + \omega_T \tau)\right],
\]

\[
s_1 s_2^* = A^2 \exp\left[j(\alpha_1^d - \alpha_2^d + \omega_T \tau)\right] + \\
F \exp\left[j(\alpha_1^d - \alpha_2^d + \omega_T \tau)\right] + \\
F \exp\left[j(\alpha_1^d - \alpha_2^d + \omega_T \tau)\right] + \\
U^2 \exp\left[j(\alpha_1^u - \alpha_2^u + \omega_T \tau)\right].
\]
\[ s_1 s_2^* = B^2 \exp \left( \frac{1}{2} \alpha_1^d - \alpha_2^d \right)^2 + U^2 \exp \left( \frac{1}{2} \alpha_1^u - \alpha_2^u \right)^2 + F \exp \left( \frac{1}{2} \alpha_1^d - \alpha_2^d \right) + \exp \left( \frac{1}{2} \alpha_1^u - \alpha_2^u \right), \quad (31) \]

\[ s_2 s_2^* = \left[ A^2 + U^2 \right] \exp \left( j \omega \tau \right) + F \exp \left( \frac{1}{2} \alpha_2^d - \alpha_2^d + \omega \tau \right) + G \exp \left( \frac{1}{2} \alpha_2^d - \alpha_2^d + \omega \tau \right), \quad (32) \]

\[ |C| = 2B^2 + 4U^2 + 2F \cos \left( \frac{1}{2} \alpha_1^d - \alpha_1^u \right) + \cos \left( \frac{1}{2} \alpha_1^d - \alpha_1^u \right) + 2 \left[ D \sin \left( \pi W \right) \right]^2 + 2G \left[ \cos \left( \frac{1}{2} \alpha_2^d - \alpha_2^d \right) + \cos \left( \frac{1}{2} \alpha_2^d - \alpha_2^d \right) \right] + 1. \quad (33) \]

where,

\[ A^2 = D^2 \sin \left( \pi W t \right) \sin \left( \pi W \right), \quad B = D \sin \left( \pi W t \right), \quad \quad (34) \]

\[ F = DU \sin \left[ \pi W t \right], \quad \text{and} \quad G = DU \sin \left[ \pi W \right]. \quad (35) \]

We can now easily determine the UDR denominator/numerator expression as,

\[ \text{DEN}(\text{UDR}) = P_d \text{NUM}(\text{UDR}) \cdot \text{D} \quad \text{and} \quad \text{NUM}(\text{UDR}) = C(0) \quad (36) \]

Figure 2: UDR vs. the desired source elevation (deg) for an ideal 2E-0T ASTSA

Figure 3: UDR vs. temporal shifter delay (microsec) for an ideal 1E-1T ASTSA

Figure 4: UDR (dB) vs. the desired source elevation (deg) for an ideal 2E-1T ASTSA

Figure 5: UDR (dB) vs. temporal shifter delay (microsec) for an ideal 2E-1T ASTSA

Figure 6: Contour plot of UDR (dB) vs. desired elevation (deg) and temporal shifter delay (microsec) for an ideal 2E-1T ASTSA

**Scenario 2: ASTSA with Frequency Independent Mutual Coupling and Channel Errors**

The received electromagnetic energy is partially reradiated from the antenna elements and this re-radiation is received from the other elements, which results in corruption of the received signal and thus affecting the correlation properties of both the desired and the undesired signal(s). This effect can be modeled as a Hermitian matrix, \( Q \) [7,8], under the assumptions that coupling effects are frequency independent, stationary,
and linear time invariant over the whole calculation interval [7,8]. Denote the distorted signal vector due to mutual coupling by \( \mathbf{x}(t) \),

\[
\mathbf{x}(t) = [x_1(t) \cdots x_A(t - B \tau)]^T.
\]  

The distorted signal vector \( \mathbf{x}(t) \) can be obtained as a linear combination of the mutual coupling matrix \( \mathbf{Q} \) and the undistorted signal \( \mathbf{r}(t) \),

\[
\mathbf{x}(t) = \mathbf{Q} \cdot \mathbf{r}(t),
\]  

where,

\[
\mathbf{Q} =
\begin{bmatrix}
q_{11} & \cdots & 0 & \cdots & q_{1A} & \cdots & 0 \\
0 & \cdots & q_{11} & 0 & \cdots & q_{1A} & \cdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
q_{1A} & \cdots & 0 & \cdots & q_{1A} & \cdots & 0 \\
0 & \cdots & q_{1A} & 0 & \cdots & q_{1A} & \cdots \\
\end{bmatrix}.
\]  

The distorted signal vector \( \mathbf{x}(t) \) is further distorted from the presence of the differences in the channel gain and phase mismatches (errors), which can be represented as constant through out the band [7,8]. If we denote with \( \mathbf{y}(t) \) the signal, which is transformed both from the presence of mutual coupling and channel errors, and with \( \mathbf{U} \) the matrix which models the differences in the channel gain and phase errors, yields,

\[
\mathbf{y}(t) = \mathbf{U} \cdot \mathbf{x}(t),
\]  

where,

\[
\mathbf{U} =
\begin{bmatrix}
u_{11} & \cdots & 0 & \cdots & 0 & \cdots & 0 \\
0 & \cdots & u_{11} & 0 & \cdots & 0 & \cdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & \cdots & u_{1A} & \cdots & 0 \\
0 & \cdots & 0 & \cdots & 0 & \cdots & u_{AB} \\
\end{bmatrix}.
\]  

Combining (40) and (41) results in,

\[
\mathbf{y}(t) = \mathbf{U} \cdot \mathbf{x}(t) = \mathbf{Q} \cdot \mathbf{U} \cdot \mathbf{r}(t) = \mathbf{V} \cdot \mathbf{r}(t).
\]  

We will again seek to minimize the UDR ratio utilizing the same minimization criteria for computing the desired set of multipliers,

\[
\mathbf{F}(\delta = 0) \cdot \mathbf{m}(t) = \lambda \mathbf{D}.
\]  

and

\[
NUM(UDR) = \mathbf{m}^H(t) \cdot \mathbf{V} \cdot \mathbf{C}(0) \cdot \mathbf{V}^H \cdot \mathbf{m}(t) \quad \text{and} \quad DEN(UDR) = P_d \left[ \mathbf{m}^H(t) \cdot \mathbf{D} \right] \left[ \mathbf{m}^H(t) \cdot \mathbf{D} \right].
\]  

where \( P_d \) is the sensitivity of the desired signal in the entrance of the antenna element. Not surprisingly the expression for the UDR ratio is obtained from,

\[
UDR = \frac{NUM(UDR)}{DEN(UDR)} = \frac{1}{P_d \left[ \mathbf{m}^H(t) \cdot \mathbf{D} \right] \left[ \mathbf{m}^H(t) \cdot \mathbf{D} \right]}.
\]  

Notice that the set of multipliers obtained from (44) will be different from those obtained from 10.

For the two-antenna element array with one temporal delay shifter scenario that is analyzed in the previous section, the mutual coupling matrix \( \mathbf{Q} \) contains only first order neighbors; therefore, it appears in the form [7,8]:

\[
\begin{bmatrix}
q_{11} & 0 & q_{12} & 0 \\
0 & q_{11} & 0 & q_{12} \\
q_{12} & 0 & q_{22} & 0 \\
0 & q_{12} & 0 & q_{22} \\
\end{bmatrix}.
\]  

The channel mismatch due to the lack of calibration is determined in accordance with [5-8],

\[
\begin{bmatrix}
u_{11} & 0 & 0 & 0 \\
0 & u_{12} & 0 & 0 \\
0 & 0 & u_{21} & 0 \\
0 & 0 & 0 & u_{22} \\
\end{bmatrix}.
\]  

The combined effect of mutual coupling and channel errors is presented with the help of matrix \( \mathbf{V} = \mathbf{Q} \cdot \mathbf{U} \):

\[
\begin{bmatrix}
q_{11}u_{11} & 0 & q_{12} & 0 \\
0 & q_{11}u_{12} & 0 & q_{12} \\
q_{12} & 0 & q_{22}u_{21} & 0 \\
0 & q_{12} & 0 & q_{22}u_{22} \\
\end{bmatrix}.
\]  

The inverse of the correlation matrix \( \mathbf{F}(\delta = 0) \) is obtained from,

\[
\mathbf{F}(\delta = 0)^{-1} = \mathbf{V}^{-H} \cdot \mathbf{C}(\delta = 0) \cdot \mathbf{V}^{-1}.
\]
This leads in computing the inverse of matrix, $\mathbf{V}$ whose determinant reads,

$$|\mathbf{V}| = q_{11}^2 q_{22}^2 u_1 u_2 u_{21} u_{22}$$

$$- q_{11} q_{12}^2 q_{22} (u_{11} u_{21} + u_{12} u_{22}) + q_{12}^4. \quad (51)$$

We will then seek to invert matrix $\mathbf{V}$ in accordance with,

$$\mathbf{V}^{-1} = \frac{1}{|\mathbf{V}|} \begin{bmatrix} v_{11}^{-1} & v_{13}^{-1} & 0 \\ 0 & v_{22}^{-1} & v_{24}^{-1} \\ v_{13}^* & 0 & v_{33}^{-1} \\ 0 & v_{24}^* & 0 \end{bmatrix}, \quad (52)$$

where,

$$v_{11}^{-1} = q_{22} u_{21} \left( q_{11} q_{22} u_{12} u_{22} - \overline{q_{12}^2} \right), \quad (53)$$

$$v_{22}^{-1} = q_{22} u_{22} \left( q_{11} q_{22} u_{11} u_{21} - \overline{q_{12}^2} \right), \quad (54)$$

$$v_{33}^{-1} = q_{11} u_{11} \left( q_{11} q_{22} u_{12} u_{22} - \overline{q_{12}^2} \right), \quad (55)$$

$$v_{44}^{-1} = q_{11} u_{12} \left( q_{11} q_{22} u_{11} u_{21} - \overline{q_{12}^2} \right), \quad (56)$$

$$v_{13}^{-1} = -q_{12}^* \left( q_{11} q_{22} u_{12} u_{22} - \overline{q_{12}^2} \right), \quad (57)$$

$$v_{24}^{-1} = -q_{12}^* \left( q_{11} q_{22} u_{11} u_{21} - \overline{q_{12}^2} \right). \quad (58)$$

Ultimately, we want to compute the UDR expression for this scenario that is,

$$\text{UDR} = \left( P_d \mathbf{g}^H \mathbf{C}^{-1} \cdot \mathbf{g} \right)^{-1}. \quad (59)$$

Where,

$$\mathbf{g} = \begin{bmatrix} v_{11}^{-1} \exp(j \alpha_1) + v_{13}^{-1} \exp(j \alpha_3) \\ v_{22}^{-1} \exp(j \alpha_1) + v_{24}^{-1} \exp(j \alpha_2) \\ v_{13}^* \exp(j \alpha_1) + v_{33}^{-1} \exp(j \alpha_2) \\ v_{24}^* \exp(j \alpha_1) + v_{44}^{-1} \exp(j \alpha_2) \end{bmatrix} \quad \text{or}$$

$$\mathbf{g} = \begin{bmatrix} v_{11}^{-1} & v_{13}^{-1} & 0 \\ 0 & v_{22}^{-1} & v_{24}^{-1} \\ v_{13}^* & 0 & v_{33}^{-1} \\ 0 & v_{24}^* & 0 \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \end{bmatrix}. \quad (60)$$

The UDR expression can be obtained from,

$$\text{UDR} = \frac{|\mathbf{V}|^2 \mathbf{C}(0)}{P_d \delta}, \quad (61)$$

where,

$$\delta = C_{11}^{-1} g_1^* + C_{22}^{-1} g_2^* + C_{33}^{-1} g_3^* + C_{44}^{-1} g_4^* +$$

$$2 \text{Re} \left[ g_1^* C_{12} g_2 + g_1^* C_{13} g_3 + g_1^* C_{14} g_4 + g_2^* C_{23} g_3 + g_2^* C_{24} g_4 + g_3^* C_{34} g_4 \right]. \quad (62)$$

Figure 7: UDR (dB) vs. the desired source elevation (deg) for a 2E-0T ASTSA (errors modeled as frequency independent)

Figure 8: UDR (dB) vs. the temporal shifter delay (microsec) for a 1E-1T ASTSA (errors modeled as frequency independent)

Figure 9: UDR (dB) vs. the desired source elevation (deg) for a 2E-1T ASTSA (errors modeled as frequency independent)
dependent, stationary, and linearly time-invariant over any
entire calculation interval, rewrite equation (38) in
frequency domain as

The signal given by expression (66) is made available to us for further processing.

The same procedure follows for calculating the steady-state correlation matrix and the desired set of multipliers.

Due to the complexity involved, an analytical expression for the steady-state correlation matrix, for the desired set of multipliers, and for the optimum value of the UDR ratio is not presented here. We are, however, showing the computation procedure and test scenario data obtained from a Matlab simulation.

We also observe that for narrowband undesired signals, treating both mutual coupling and channel errors as frequency dependent produces the same effect as treating mutual coupling and channel errors as frequency independent.

For narrow band signals expression (66) is reduced to (42) and the same results can be obtained when mutual coupling or frequency dependent errors are applied.

Scenario 3: ASTSA with Frequency Dependent Mutual Coupling and Channel Errors

Under the assumption now that mutual coupling is frequency dependent, stationary, and linearly time-invariant over the entire calculation interval, we rewrite equation (38) in frequency domain as

\[ Y(f) = U(f) \cdot X(f), \quad (65) \]

and in time domain representation,

\[ y(t) = \int_{f_T-W/2}^{f_T+W/2} Y(\xi) \exp(j2\pi\xi t) d\xi \]
\[ = \int_{f_T-W/2}^{f_T+W/2} U(\xi) Q(\xi) R(\xi) \exp(j2\pi\xi t) d\xi. \quad (66) \]

The same procedure follows for calculating the steady-state correlation matrix and the desired set of multipliers.

For narrow band signals expression (66) is reduced to (42) and the same results can be obtained when mutual coupling or frequency dependent errors are applied.

Summary and Conclusions

Overall, we observe that a better suppression of the undesired narrowband signals can be obtained when both spatial and temporal degrees of freedom are utilized effectively. Such is the case of the ASTSA, which appears to provide an undesired signal suppression of 80 dB.

For narrowband signals, frequency dependent and/or independent errors produce the same signal distortion; therefore, their effect results in the same undesired signal suppression.

It appears that the ASTSA compensates adaptively the undesired effect of the frequency dependent and/or independent mutual coupling and channel errors, while suppressing undesired narrowband signals; hence, eliminating the need for extensive calibration of the ASTSA.

Future work will explore the performance of this approach in a variety of interference scenarios.
References


