PAPER No. 14.

ZERO ERROR OF THE MODEL MRA 101 TELLUROMETER.

A.J. Robinson

SUMMARY. The Model MRA 101 Tellurometer was received by the Department of Surveying, University of New South Wales early in 1966. Previous models of the tellurometer had exhibited a zero error that was cyclic and dependent on the A reading and investigations have shown that the zero error of the Model MRA 101 is of a similar form. Application of this cyclic zero error and a constant zero error are compared with results using the zero error value recommended by the manufacturer.

1. INTRODUCTION.

In February, 1966, the Department of Surveying at the University of New South Wales received three tellurometers, Model MRA 101, Serial numbers 110, 113 and 114. These instruments at the time were the latest addition to the Tellurometer range. No description of the instrument will be given in this paper as the instrument has been described in previous publications (TELLUROMETER, 1965; SAASTAMOINEN, 1967; WEBLEY, 1965).

Tellurometer models MRA 1 and MRA 2 with cathode ray oscilloscope readouts and model MRA 3 with a digital readout had exhibited a zero error that was cyclic and dependent on the A reading. (BEDSTED, 1962; LILLY, 1963; YASKOWICH, 1965; BOSSLER and LAURILA, 1965).

The model MRA 101 Tellurometer has a digital readout in metres. The manufacturer recommended a constant zero correction of minus 0.30 metres and also suggested the instruments be calibrated. As the earlier models had shown a cyclic zero error and as the manufacturer had recommended calibration, it was decided to examine the instruments for zero error.

2. THE TEST AREA.

The test area sought was a large open field of level topography, preferably with controlled public access. The intention was to measure a base line of about 600 feet, free from obstruction on both sides of the base. This area had to be close to Sydney because of difficulties of transporting staff and equipment.

very useful in the setting up and plumbing at each station.

Before the tellurometer measurements were taken the carrier frequency and modulation frequencies of the instruments were measured by the School of Electrical Engineering at the University of New South Wales. The pattern frequencies were measured using a Hewlett-Packard Frequency Counter. The carrier frequencies were measured using a Hewlett-Packard 3 cm. wave meter type X532B. The results of these measurements are shown in Appendix 1.

3.2 Psychrometer. The psychrometer used to measure wet and dry bulb temperature was a mechanically operated "Assmann's Aspiration Psychrometer." The thermometers are graduated to one degree centigrade and readings were estimated to 0.2°C. The thermometers were calibrated by the National Standards Laboratory, Sydney, the largest corrections to the observed temperature being -0.5°C at 0°C and +0.5°C at 25° and 35°C.

3.3 Barometer. The barometer used to measure atmospheric pressure was a 'Baromech' aneroid barometer (contact type). This barometer measures atmospheric pressure to 0.5 mm of Hg with estimation to 0.1 mm. For a full report of this type of instrument see ALLMAN (1968).

4. THE EXPERIMENT.

The A reading on the model 101 tellurometer can vary from 0 to 10 metres. In order to distribute the A reading over this range, the ten base lines as described in Section 2 were measured. All observers are staff members from the Department of Surveying, University of New South Wales, and are experienced tellurometer observers. One measurement of a base line consisted of the mean of two determinations of the distance, one with instrument No. 113 as master, the other with instrument No. 110 as master. For each measurement, twenty fine readings were taken at ½ carrier tune settings.

One of the modifications mentioned in section 3.1 consisted of a small sighting slot cut in the metal base of the instrument. As no optical plumbing system was provided on the instrument or tripod, the instruments were centred by means of a theodolite set at 90° to the line. The instruments were directed towards each other by sighting through the small slot and then a point on the screw holding the instrument on the tripod was plumbed by means of the theodolite. This process was repeated until the other instrument was in the centre of the field of view of the slot, and the tripod screw was centred over the ground mark.

To eliminate any systematic error of centring, the base lines were measured five times. The instruments were not used consistently at the same end of the base lines and the measurements of each line were made in as varied weather conditions as possible. During the measurements the temperature (dry bulb) ranged from 19°C to 38.5°C, pressure range was from 755.0 mm. Hg. to 767.0 mm. Hg., and general weather varied from hot, humid conditions to cold, wet and windy conditions.

5. OBSERVATIONS.

5.1 Meteorological Observations. Meteorological observations were taken before and after each set of fine readings at one end with the equipment described in 3.2 and 3.3.

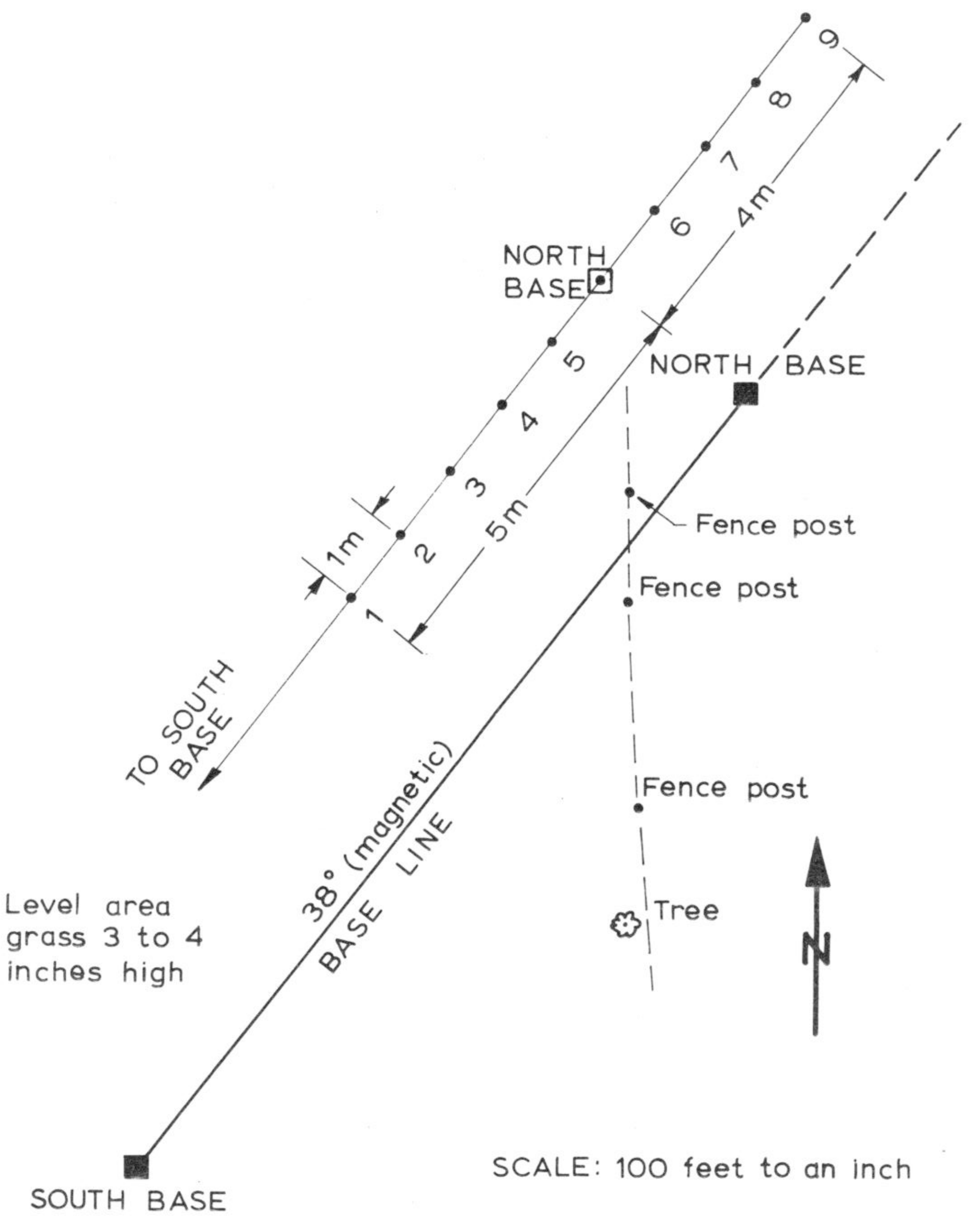

FIG. 14.1 DIAGRAM OF BASE LINE – WARWICK FARM

The Australian Jockey Club offered the use of the No. 2 Polo Field at Warwick Farm, which is about 15 miles west of Sydney. This area (Fig. 14.1) is relatively level and is covered with grass about 3 to 4 inches high. Public access is controlled and the chosen base line is free from obstruction on both sides with the exception of a tree and three fence posts about 3 feet high. The base line has a magnetic bearing of 38^{o} and is at an elevation of 15 feet above Standard Datum.

The terminal points, South Base and North Base are stainless steel pins set in a concrete block about 2 feet x 2 feet x 2 feet cast in situ and covered with a cast iron cover box. The base line was measured by the New South Wales Department of Lands. The equipment used for the measurement was Watts Standard Traversing Equipment including a standardised 200 foot x 1/8 inch Invar Band. The weather conditions for the measurement were calm, overcast, with very light rain and a temperature range of 72^{o} to 75^{o}F.

The length between the terminal points is 599.390 feet (183.694 metres) and is considered to be accurate at least to within 1 part in 50,000.

At the northern end of the base line nine additional marks were placed (Fig. 14.1 inset), approximately one metre apart, to give a total of ten points which, when used in conjunction with South Base, gave ten base lines.

These additional points were centre punch marks in tacks in 3 x 3 x 16 inch wooden pegs driven flush with the surface.

Each time the nine auxiliary points were used the distances between them and North Base was measured. This precaution was taken in case any of the pegs moved. The measured distances were then added or subtracted to the base length in order to obtain the length of the auxiliary base line. The greatest difference between measurements from the same peg to North Base on different days was two millimetres.

3. THE EQUIPMENT.

3.1 Tellurometer. The model MRA 101 has a digital readout in metres, giving a variation of 0 to 10 metres with a vernier provided to read directly to centimetres.

The carrier frequency can be varied over the range of 10,050 Mc/s to 10,450 Mc/s, the particular setting being shown by a number (0, 1, 2, 3, 4, 5, 6, 7) in a window above the carrier tune control knob.

These numbers are etched on a circular disc (see Fig. 14.2) and, as no pointer was provided, it was difficult to set a reading on the instrument. A modification was made to this part of the instrument. The graduations on the dial were increased to show a line through the numbers and a 'dot' was added to show ¼ carrier tune settings (Fig. 14.2). The number 0 was changed to 8. A knife edge pointer was placed behind the window and in front of the scale so that a parallax free setting could be made on the graduated dial. This modification was made so that the remote could set the same carrier tune setting throughout all measurements in the experiment.

During the initial setting up of the instrument, difficulty was experienced in pointing the instruments towards one another. A further modification was made in order to overcome this difficulty. A small sighting slot was cut in the metal plate, mounted on the bottom of the fibreglass case, in the direction of the antenna. Through this slot the other instrument could be viewed. This modification proved

The refractive index was then calculated from a nomogram, the mean of the two refractive indices being accepted as the refractive index for the measurement. The range in the refractive index was from 1.000339 to 1.000400. Because the base lines were of the order of 183.00 metres, variation in meteorological observations had little effect on the distance, the greatest correction being 14 millimetres to a line of 186 metres.

5.2 Tellurometer Observations. As mentioned in Section 4, twenty fine readings, covering the whole carrier frequency range, were taken with each instrument as master. The remote carrier tune settings for these twenty fine readings were the same for each measurement giving true repetitive measurements. When the instrument was centred by means of the theodolite and the small sighting slot, it was found the instrument could be reset to within about 3 millimetres so a conservative estimate of the standard deviation of centring would be about 2 mm.

6. GROUND SWING.

A typical ground swing curve is shown in Fig. 14.3. The average range of ground swing for the 100 measurements was 8 centimetres, the largest being 12 cms., the smallest 5 cms. The final value for the A reading was taken as the arithmetic mean of the 20 fine readings.

7. RESULTS.

The tellurometer measurements of the ten base lines and the true value for these bases are shown in Table I. The tellurometer length for the individual base lines is found from the mean of the five measurements minus the manufacturer's value for zero correction, 0.30 metres.

The difference between the tellurometer length and the true length gives an additional zero correction which is shown in the table. The last entry in the table is the standard deviation of a single tellurometer measurement, which is between ± 0.015 and ± 0.033 m.

Fig. 14.4 shows the additional zero correction for a particular line graphed against the tellurometer A reading for that line. It appears from this graph that the additional zero correction is cyclic and depends on the A reading.

A least squares curve was fitted to the additional zero correction results and is shown on Fig. 14.4 with the original observations.

The curve has the form:

$$\text{Zero correction} = b_1 + b_2 \sin \left(2\pi\frac{A}{10} + b_3\right)$$

where b_1, b_2 and b_3 are constants and A = A reading in metres.

The values of the constants substituted in the equation give:

Additional zero correction = $0.0460 + 0.0096 \sin \left(2\pi\frac{A}{10} + 68^\circ\!.8\right)$ metres. The additional correction constant of 4.6 cms indicates that the manufacturer's value for the zero correction cannot be accepted, and that further calibration is necessary. Cyclic error, of amplitude approximately 1 cm, is present.

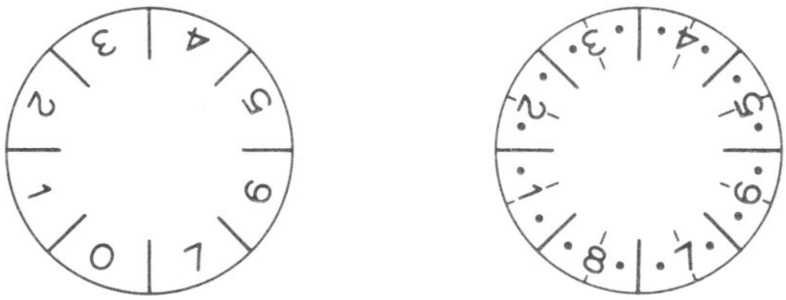

FIG. 14.2: MODIFICATIONS TO CARRIER TUNE DIAL. LEFT ORIGINAL RIGHT – AS MODIFIED

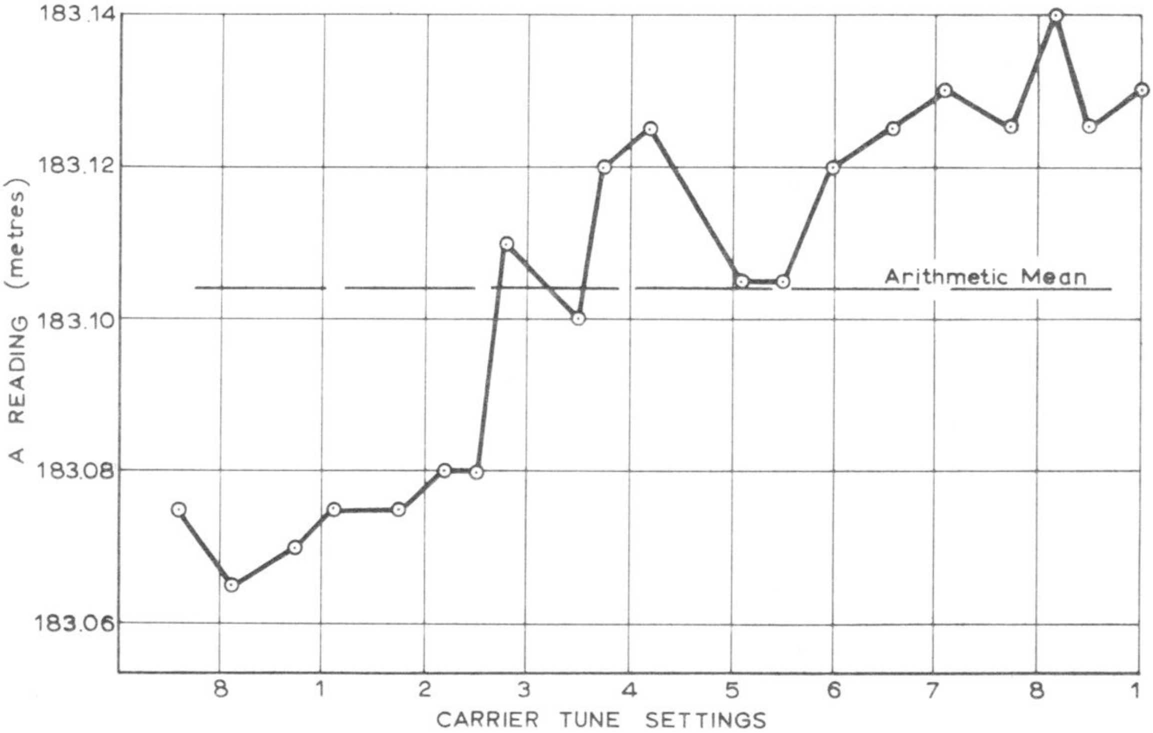

FIG. 14.3: TYPICAL GROUND SWING CURVE

TABLE I.

Tellurometer measurements for determination of cyclic error - metres.

Base Line No.	1	2	3	4	5
Mean Tellurometer length	178.055	179.044	180.058	181.053	182.042
Mean - 0.30	177.755	178.744	179.758	180.753	181.742
True length	177.704	178.700	179.700	180.695	181.694
Zero Correction	- 0.051	- 0.044	- 0.058	- 0.058	- 0.048
Standard deviation of a single observation	± 0.029	± 0.033	± 0.026	± 0.027	± 0.028

Base Line No.	Base	6	7	8	9
Mean Tellurometer length	183.042	184.032	185.021	186.015	187.009
Mean - 0.030	182.742	183.732	184.721	185.715	186.709
True length	182.694	183.685	184.684	185.680	186.675
Zero correction	- 0.048	- 0.047	- 0.037	- 0.035	- 0.034
Standard deviation of a single observation	± 0.023	± 0.024	± 0.022	± 0.015	± 0.018

8. TEST OF CYLCIC ZERO CORRECTION.

In order to test the above determination of the additional cyclic zero correction the results of a second experiment were used. These results are from tellurometer measurements observed as described above, taken over the 183 metre base line with instrument heights varying from 1.22 metres (4 feet) to 30.48 metres (100 feet). The change in instrument height was accomplished by using two tubular steel towers, each 6 feet square and 96 feet high as shown in Fig. 14.5. Eight landings, twelve feet apart, were provided in the towers. Each landing had a deck for the instrument and a separate deck for the observer.

The towers were guyed at the four corners at 30, 60 and 90 feet from the ground. The guys made the towers very stable, for when observing, the tellurometers were plumbed and the greatest movement during an observation was 5 millimetres, which occurred during strong wind gusts.

The method of plumbing made use of a theodolite set 90° to the base line and a scale that was set over the ground mark. The tellurometer was sighted with the theodolite and the scale was then observed, giving a correction that could be applied to the base line length. An observer was present on the ground while all tellurometer measurements were taken. By plumbing the tellurometers in this manner the true distance for each measurement was known. All combinations from one tower to the other including repeat measurements to equal levels were observed.

The 53 lines measured gave a range in A reading from 3.0 to 5.4 metres. As the true length of the line was known, a study of the effect of zero correction could be carried out. The standard deviation of the measurements was calculated, using three different zero corrections. In the first method the tellurometer length was determined by applying the manufacturer's value for zero correction. In the second method, the additional cyclic zero correction as determined from Fig. 14.4 was applied to the tellurometer lengths. In the third method a constant additional zero correction of 0.046 metres was applied to the tellurometer lengths. This value of 0.046 metres is the mean value of the additional cyclic zero correction.

TABLE II.

Values of the standard deviation of the measurements calculated by three methods.

	Method 1.	Method 2.	Method 3.
Total zero correction	0.300 m.	0.300 m. + cyclic value	0.300 + 0.046 m.
Standard deviation	±0.066 m.	±0.029 m.	±0.030 m.

Table II shows the effect of the application of a cyclic zero correction. The standard deviation is reduced from 0.066 metres to 0.029 metres.

It must be stressed however, that applying an additional constant zero correction which is the mean of the values found during the cyclic determination gives no improvement in the standard deviation found from that calculated using the cyclic value.

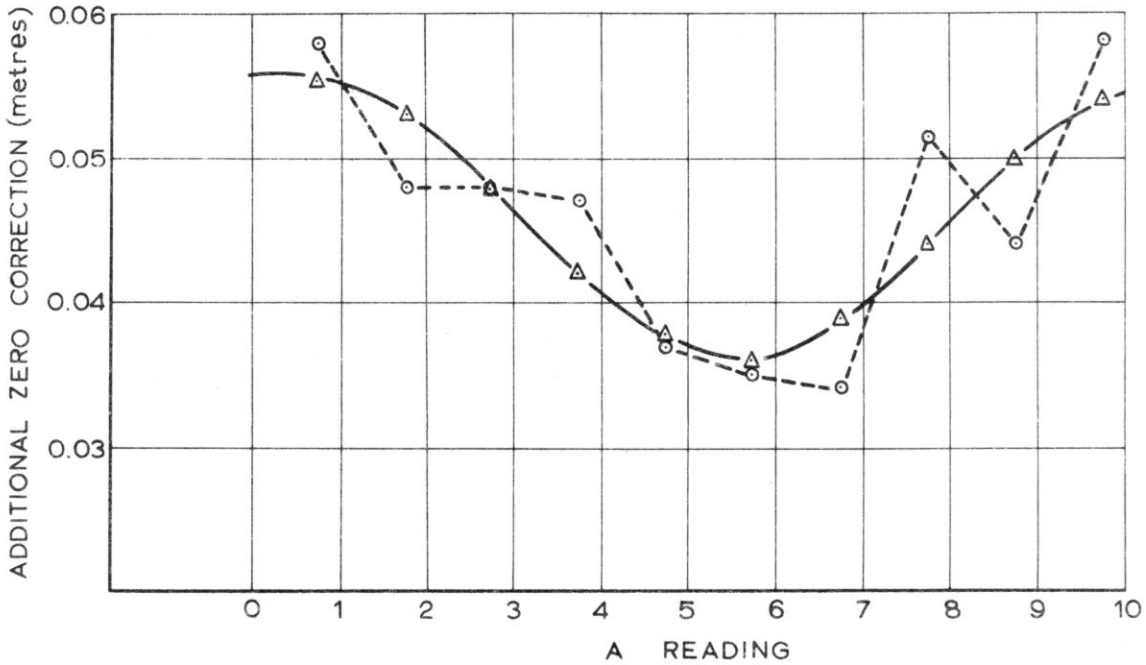

FIG. 14.4:- ADDITIONAL ZERO CORRECTION SHOWING CYCLIC EFFECT AND LEAST SQUARES FIT OF SINE CURVE : $4.60 + 0.96 \sin (2\pi \frac{A}{10} + 69°)$

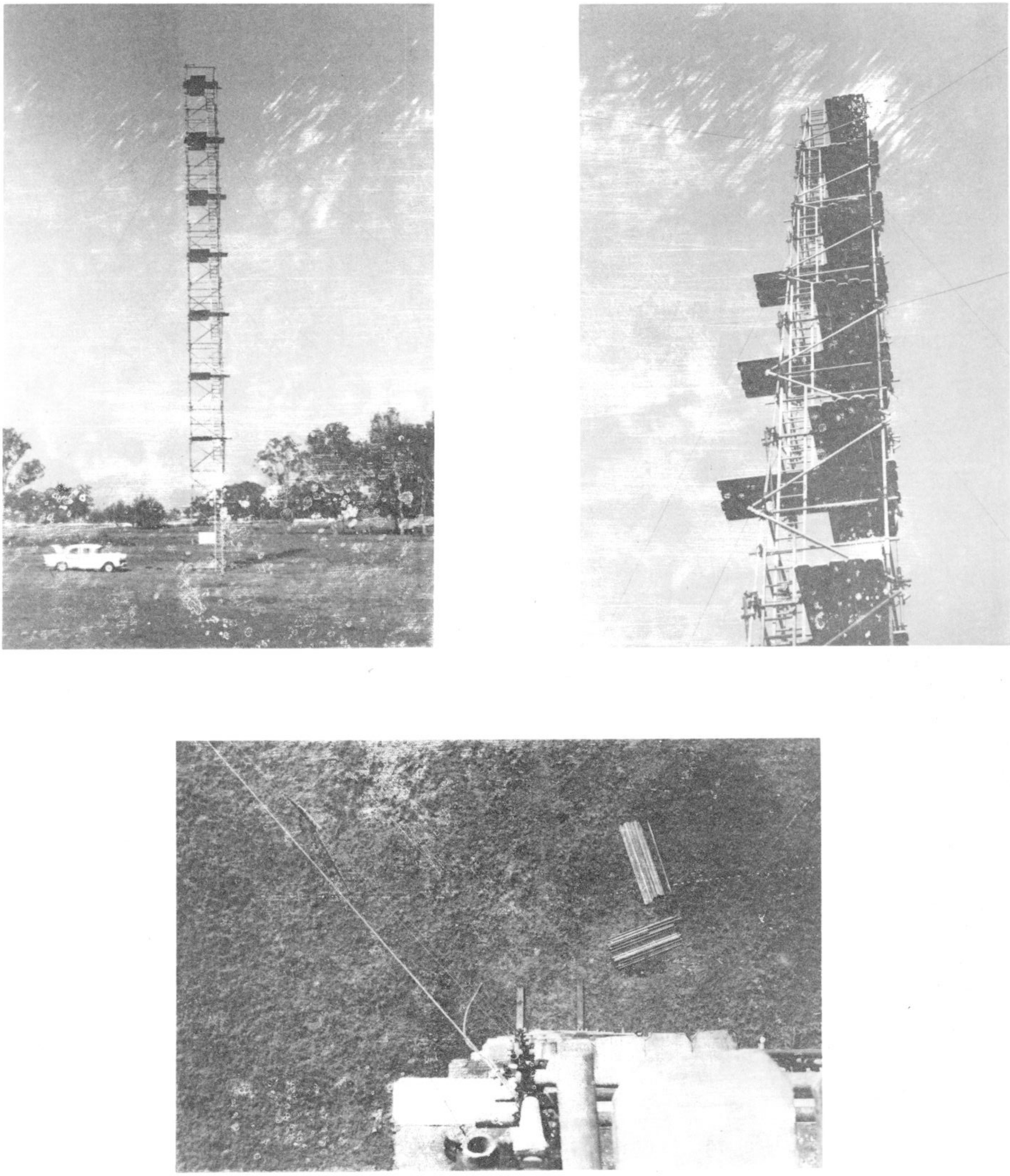

FIG. 14.5.

9. CONCLUSION.

The results show that the zero correction of the Model MRA 101 tellurometer is cyclic and depends on the A reading. No improvement in accuracy is gained by application of this cyclic zero correction provided the zero correction used is found from an instrument calibration.

10. ACKNOWLEDGEMENT.

I would like to thank the staff members of the Department of Surveying at the University of New South Wales for their assistance with the field work.

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APPENDIX 1.

Pattern Frequencies.

Pattern	Frequencies. (Mc/s) No. 110	No. 113	Manufacturer's Values
A Master	7.492 381	7.492 380	7.492 377
E Master	5.993 903	5.993 896	5.993 902
D Master	7.342 532	7.342 533	7.342 529
C Master	7.477 390	7.477 389	7.477 392
B Master	7.490 876	7.490 847	7.490 879
B Remote	7.489 875	7.489 860	7.489 879
C Remote	7.476 384	7.476 390	7.476 392
D Remote	7.341 529	7.341 537	7.341 529
E Remote	5.992 906	5.992 899	5.992 902
A Remote	7.493 388	7.493 335	7.493 377
Ref Remote	7.491 386	7.491 379	7.491 377

Carrier Frquencies (Mc/s)

Carrier tune setting	Frequency		Carrier tune setting	Frequency	
	No. 110	No. 113		No. 110	No. 113
7.5	10.055	10.054	5.0	10.239	10.240
8.0	.067	.064	5.5	.260	.263
8.5	.078	.093	6.0	.278	.281
1.0	.091	.098	6.5	.287	.304
1.5	.107	.110	7.0	.321	.324
2.0	.132	.126	7.5	.338	.339
2.5	.155	.143	8.0	.360	.358
3.0	.166	.161	8.5	.378	.381
3.5	.180	.183	1.0	.392	.398
4.0	.198	.204	1.5	.414	.415
4.5	.219	.238	1.8		.425
5.0	.239	.240	2.0	.432	

PAPER NO. 15.

ON THE OPTIMUM PHASE-COMPARISON RADAR SIGNAL

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SUMMARY. Given the maximum distance to be measured and the maximum allowable relative error, we state that the optimum phase-comparison radar signal is the signal which allows the distance measurement by employing the minimum energy.

Starting from the radar-signal theories and keeping in mind that all the signals considered should exhibit the same precision and the same non-ambiguous range, we have synthesized the theoretical optimum signal, according to the above meaning of optimum. One of the main characteristics of the optimum signal is the coincidence between the measuring signal and the transmission signal.

Serious problems in the practical use of this signal have forced us to look for pseudo-optimum signals which can be derived from the optimum signal, by modulating a suitable carrier in different ways.

Several pseudo-optimum signals have been investigated, their difference being principally restricted to the relations between the spectra of the measuring and the transmission-signal.

1. INTRODUCTION.

All systems which make use of any kind of electromagnetic-waves to obtain distance measurements for geodetic purposes, can be really regarded as particular radar systems, by far the most important characteristic of which is a very high accuracy. Therefore it is a rather obvious idea to take advantage of the general radar-signal theories, with a view to improving the intrinsic performance of the EDM systems.

From the radar theories it appears that the choice of a radar signal for any particular application is always subject to a certain compromise among several constraints, which in general arise from the need to obtain different kinds of information from the same signal. In our case these constraints are typically the non-ambiguous range and the distance-precision, while any other requirements, like doppler measurements, distance and doppler resolution can be disregarded as inessential. The analysis of geodetic signals can thus appear rather easy in comparison to other applications.

The general treatment of the radar signal considers the signal directly at the input of the receiver, without taking into account the reflectivity of the target (which may be a transponder, too) and of the characteristics of the transmission medium, including the distance to be measured. The main characteristics of the signal are frequency and phase spectra and also the signal-to-noise ratio of its energy.

The higher the signal-to-noise ratio, the better the quality of the measurements which can be obtained from a received signal. The signal-to-noise ratio in turn is maximum for a receiver which is matched to the signal itself, i.e. a receiver, whose transfer-function is proportional, apart from a constant, to the complex conjugate of the frequency spectrum of the signal. In what follows we shall assume this condition as fulfilled.

2. THE PHASE METHOD.

The first requirement for a significant determination of distance is the elimination of the range ambiguity. In fact, the phase-difference between the transmitted and the received signal (at a frequency f), due to the propagation time t, being expressed as:

$$\phi = 2\pi ft. \tag{1}$$

can be unambiguously measured at the receiver, provided that $\phi < 2\pi$. Otherwise an indeterminacy of multiple of 2π remains. Accordingly, in order to avoid any range ambiguity in the distance measurement one is forced to set the condition:

$$f \cdot t < 1 \tag{2}$$

Thus, given the maximum distance to be measured L_{max}, i.e. the maximum propagation time $t_{max} = 2L_{max}/n.c$, the condition (2) represents an upper limit for the frequency f.

But using only one frequency, the need of a very high precision, as for geodetic purposes, would require a tremendous amount of energy, as will appear from the following arguments.

Let us consider the absolute error (measured in terms of time) which is connected with the phase-measurement. In the case of a single-frequency signal, this time error, as derived from the Woodward's theories, can be expressed as:

$$\Delta t = \frac{1}{2\pi f\sqrt{R}} , \tag{3}$$

where f is the frequency of the signal and R its signal-to-noise ratio. One can observe that the expression (3), through the relation (1), leads to a very simple and meaningful relation between the phase-error $\Delta\phi$ and R, namely:

$$\Delta\phi = \frac{1}{\sqrt{R}} \tag{4}$$

By dividing side-by-side the relations (4) and (1), one obtains an expression for the relative error:

$$\delta = \frac{\Delta\phi}{\phi} = \frac{1}{2\pi ft\sqrt{R}} , \tag{5}$$

which in turn recalling condition (2), leads to the significant inequality:

$$R > \frac{1}{(2\pi\delta)^2} \tag{6}$$

From this inequality it follows that, because of the strict requirements of precision for geodetic distance measurements, the signal-to-noise ratio R should assume, in the case of a single-frequency signal, values which are really high. Inversely, the precision $1/\delta$ which could be obtained in the distance measurements with a given amount of signal energy at the input of the receiver would be comparatively very poor.

On the other hand, it appears from Eq. (3) that increasing the frequency, rather than the energy, would be convenient for the reduction of the absolute error.

The above considerations suggest the opportunity of employing more complicated signals, i.e. more than one frequency. In this way the requirements of precision, which call for higher frequencies, can be separated from those of the ambiguity resolution, which on the contrary impose an upper limit on the frequency.

3. THE MEASUREMENT SIGNAL.

The main condition which couples in pairs the results of the phase measurements which come from the different frequencies of the signal, may be stated as follows: every measurement obtained at one frequency must eliminate the ambiguity which is left by the measurement (intrinsically more precise) obtained at the next higher frequency.

Let us call f' and f'' two contiguous frequencies of the signal spectrum and $\Delta t'$ and $\Delta t''$ the absolute time-errors resulting from the corresponding frequencies. The following condition must be met:

$$\Delta t' + \Delta t'' \leq \frac{1}{2f''} \quad , \tag{7}$$

which can be called the "overlapping condition". In this case we say that the measurements "overlap".

Recalling relation (3) this condition becomes:

$$\frac{1}{2\pi f'\sqrt{R'}} + \frac{1}{2\pi f''\sqrt{R''}} \leq \frac{1}{2f''} \tag{8}$$

which in turn leads to the Eq. (9)

$$\frac{K''}{\sqrt{R'}} + \frac{1}{\sqrt{R''}} = h\pi \quad , \tag{9}$$

where is $K'' = f''/f'$ and $h(\geq 1)$ represents something like a safety-coefficient for the overlapping.

Eq. (9) gives a basic relation between frequencies and energies of the pairs of adjacent spectral lines. The overlapping condition eliminates the propagation of the errors due to the measurements obtained by the various signal frequencies; in fact every lower frequency only serves to solve the ambiguity which is left by the next higher frequency. Accordingly the relative error of the ultimate distance measure is given by:

$$\delta = \tau_M f_1 \quad , \tag{10}$$

where τ_M is the absolute error due to the M-th frequency (the highest) and f_1 is the lowest frequency.

From the operative point of view the frequencies of the signal spectrum can be transmitted and employed separately or all together but from the theoretical point of view the two methods are perfectly equivalent to each other, because in any case one must designate by signal the complete frequency spectrum as a whole. Besides, the signal energy is equal to the addition of the energies of every spectrum line, as shown by the following equation:

$$R_t = \sum_{1}^{M}{}_i R_i \quad , \qquad (11)$$

where R_i represents the energy of the general spectrum line relative to the noise energy, or signal-to-noise ratio. The parameters which characterize the signal are the frequencies and the energies of the spectrum lines and their number, while its requirements in the case of geodetic distance-measurements, are the non-ambiguous range and the relative precision of the measurement. With a given amount of signal energy at the receiver input, the performances which can be obtained through the phase measurement depend greatly on the signal parameters.

It is then worth looking for the best signal-performance through a suitable choice of its parameters. The synthesis of this signal which will be called "optimum", can be done by setting up and working out the following system of equations which comprises 2M + 1 equations:

$$\left\{\begin{array}{lll} \dfrac{K_j}{\sqrt{R_{j-1}}} + \dfrac{1}{\sqrt{R_j}} = h\pi \quad , & j = 2 \div M & \\ \delta = \dfrac{f_1}{2\pi\sqrt{R_M}\cdot f_M} \quad , & & \\ R_t = \sum_i R_i \quad , & i = 1 \div M & (12) \\ \dfrac{\partial R_t}{\partial K_j} = 0 \quad , & & \\ \dfrac{\partial R_t}{\partial M} = 0 & & \end{array}\right.$$

Namely:

M-1 equations expressing the overlapping condition between as many pairs of continuous frequencies. See Eq. (9)

1 equation imposing the relative precision required in the ultimate distance measurement. See Eq. (10).

1 equation giving the total energy of the signal. See Eq. (11).

M-1 equations representing the conditions for the optimum signal as function of M-1 frequencies. (recall that the lower frequency f_1 is a datum of the problem through the maximum non-ambiguous range). See Eq. (2).

1 equation representing the condition for the best signal as function of the number M of the spectrum lines.

We can observe that since M is one of the unknowns of the system, knowledge of the power of the system itself is a priori impossible. This fact makes the analytical solution for the optimum signal difficult.

We can also observe that the number of the spectrum lines as well as the number of system equations must be an integer, while the value of M, as determined through the analytical solution of the system, could also be a non-integer. Hence this analytical solution could only be exactly determined for several discrete values of the relative precision $1/\delta$.

The following method of solution, consisting of more than one step, has seemed to us to be the more practical: first to disregard the last equation $\partial R_t/\partial M = 0$, secondly to attribute to the number M one integer value, third to solve the system, obtaining one signal and the corresponding value of $R_t(M)$, finally to increase the value of M and to repeat the calculation. In such a way we could single out for every M the best signal. By comparing the various $R_t(M)$ we could easily find the optimum value of M and then the optimum signal. In Fig. 15.1 the values of $R_t(M)$ corresponding to a relative precision $1/\delta = 10^6$ are plotted against M. It appears from the plot that R_t strongly depends on M, when its value is low, but the curve flattens out for larger values of M.

Similar plots have been drawn up for different values of the precision, namely for $1/\delta = 10^3, 10^4, 10^5$. The values of the optimum M are 18, 8, 11, 15 in the four cases respectively. As we have observed before, none of the corresponding signals is the exact analytical solution of the system. In spite of that, the values of frequencies and energies so obtained disagree very slightly with the solutions of the following equations, which can thus be regarded as the true system solution:

$$\left\{\begin{aligned} R_1 &= \frac{2}{3} \quad , \\ R_j &= 1, \quad (j = 2 + M) \ , \\ K_2 &= \sqrt{\frac{2}{3}}\,(\pi - 1) \ , \\ K_w &= \pi - 1 \quad , \quad (w = 3 + M) \quad , \\ M &= 1 + \frac{\log(1/\delta) - \log(\pi\sqrt{8/3})}{\log(\pi - 1)} \\ R_t &= M - \frac{1}{3} \end{aligned}\right\} \qquad (13)$$

Apart from the value of the ratio $K_2 = f_2/f$, the values of any other K_j are little different from 2. (See Fig. 15.2).

On the other hand it may be more convenient, from a practical point of view, to synthesize a signal whose frequencies are harmonics of one another, rather than be tied by a real number.

Fortunately, the total energy of these harmonic signals is very little higher than the total energy of the optimum signal.

In fact the harmonic signal, with a value of the ratio $K = f_{j+1}/f_j = 2$, calls for M = 19 lines and $R_t = 17.33$, while the harmonic signal, with K = 3, calls for M = 12 lines and $R_t = 19.45$. The comparison of these values of R_t with the value $R_t = 16.6$ of the optimum signal proves the above assertion.

A final comment on the phase-comparison signals is worthwhile. Most of the geodetic instruments which make use of the phase method, employ three or, at the most, four frequencies. Looking at the plot of Fig. 15.1 one can immediately realize that the best signals, composed of 3 or 4 frequencies, ask for an energy about two orders of magnitude higher than that of the optimum signal.

4. THE RF SIGNALS.

The frequency range of the optimum signal comes out so broad (f_1 = 150 Hz, f_M=23.8 MHz in the given example), that the direct transmission of this signal may be really impracticable. In order to overcome this inconvenience it is necessary to translate the whole spectrum in a proper frequency range, consequently reducing the proportional difference between f_M and f_1. While several translation methods can be employed, it appears through a more careful analysis, that the best translation method consists of the single side-band modulation of the measurement signal. Any way, whatever the translation method actually chosen, the measurement signal, at the receiver output, must bring in the same information content as it would do in the case of a direct transmission. This fact calls for the transmission of the carrier together with the true transmission signal.

In fact only in this case, mixing every transmission frequency f_{ip} with the carrier frequency f_p, one obtains the corresponding measurement frequently -

$$f_i = f_{ip} - f_p, \tag{14}$$

and therefore the phase to be measured:

$$\phi_i = \phi_{ip} - \phi_p \quad , \tag{15}$$

The total energy of the transmission signal, then, is higher than that of the measurement signal owing to the carrier energy.

But this is not the only reason every frequency of the transmission signal must be more energetic than that of the measurement signal, i.e. $R_{ip} > R_i$, as appears from the following argument.

Recalling equation (15), one obtains for the absolute phase errors:

$$\Delta\phi_i = \sqrt{\Delta\phi_{ip}^2 + \Delta\phi_p^2} \quad ,$$

which in turn, through equation (4) gives the relation

$$\frac{1}{R_i} = \frac{1}{R_{ip}} + \frac{1}{R_p} \quad ,$$

or:

$$R_{ip} = \frac{R_p . R_i}{R_p - R_i}$$

Therefore it is necessary that $R_p > R_i$ and $R_{ip} > R_i$.

Whichever method for the translation of the measurement signal is chosen, the resulting transmission signals are slightly different from one another, so that in any particular case one could determine the best through calculations of more or less difficulty.

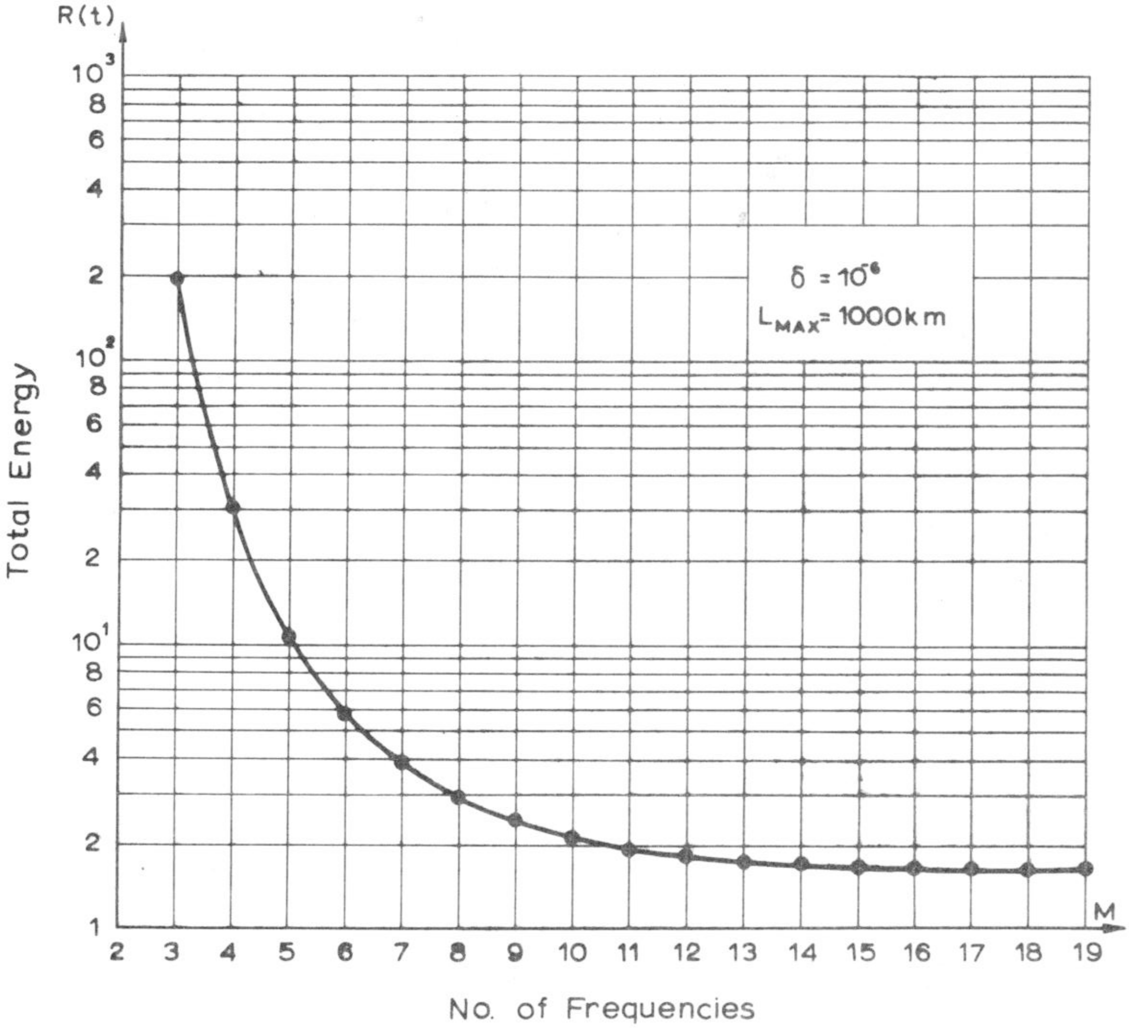


FIG. 15.1 TOTAL ENERGY R(t) AS A FUNCTION OF NO. OF FREQUENCIES M

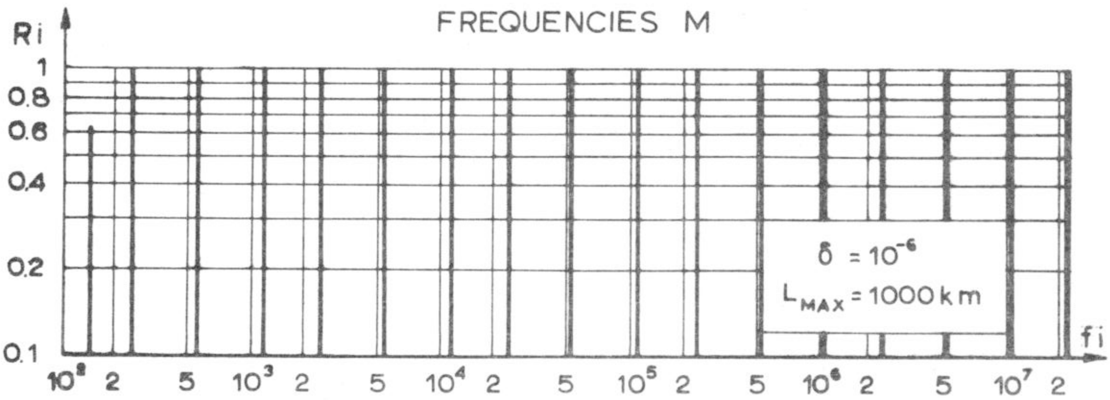


FIG. 15.2 SIGNAL ENERGY Ri AND FREQUENCY fi

We have done the complete analysis of three different transmission signals, the difference among them being only limited to the frequency-translation method.

From this analysis it appears that the equivalent measurement signals differ very little from each other, so that we can conclude that, while the optimum measurement signal must be chosen through theoretical considerations, the choice of the modulation method might be made only through practical considerations.

ACKNOWLEDGEMENT: The authors wish to thank Mr. Ferdinando Pasqualetti for his kind help in developing the calculations on the electronic computer.

PAPER NO. 16.

AUSTRALIAN EXPERIENCE WITH TELLUROMETER MRA4

O.J. Bobroff

SUMMARY. General comments are given on the behaviour of Tellurometer MRA4 in the field with some results of zero correction determinations and performance comparisons with MRA1 over long distances.

A great deal of valuable work has been done with Tellurometers and it may be said that in Australia these instruments have made the completion of our Geodetic Survey possible in a reasonably short time. Since the introduction of the first model in the late fifties many improvements and refinements have been made, culminating in the present model MRA4, and it is proposed to discuss this model from the viewpoint of the user.

Three units of MRA4 have been in use by the Division of National Mapping for over a year and some comments can now be made:-

(a) After some minor teething troubles, no repairs have been necessary in the field, the mechanical and electrical reliability exceeding that of any previous model used, i.e. models MRA1, 2 and 3.

(b) Two units have held their crystal frequency calibration well, varying only one or two cycles/month in use, while the third has shown a regular downward drift of about 7 cycles/month. This will be investigated when the instruments return from the field.

(c) Experience with previous models had shown the importance of maintaining battery voltage within specific limits to ensure acceptable crystal frequencies, but early tests with the MRA4 indicated that no variation in the frequency occurred with the varying of input voltages from 14 volts down to 4 volts, at which point the instrument ceased to operate. This is probably due to the superior components now used and to the stabilised power supply.

(d) No real difficulty was experienced in aligning instruments even on long lines, providing the stations were on well defined mountain peaks or identifiable points. The narrow beam made preliminary visual contact mandatory in cases where the location of the stations was uncertain, since compass bearings have not always been adequate. For this purpose, automotive driving lamps fitted with sights and quartz iodine bulbs were used, being more convenient than either heliographs or the signalling lamps commonly employed.

(e) In the field the instruments were mounted on Wild T3 theodolite tripods carrying Wild T2 tribrachs which accepted the tilting and aligning heads supplied by the makers. Centring was by the optical plummet in the T2 tribrach. While no trouble was experienced with this auxiliary equipment it is felt that some improvement is necessary, mainly to improve the rigidity of the system, which vibrates perceptibly in high winds and when tuning controls are operated.

(f) It is not possible to quote a maximum range at this stage as the maker's figure of 50 kms has been exceeded on many occasions. One successful measurement was over 90 kms.

(g) It has been noticed that measurements along narrow lanes through timber have been very consistent, with little indication of signal attenuation.

(h) A series of measurements of the same line taken throughout the day at hourly intervals revealed a variation in length which warrants fuller investigation. There is some indication of a "best-time-of-day" for measuring just as there is for horizontal and vertical angular measurements.

In addition to the specific field programme, some test measurements were carried out at three triangulation base lines laid down by the Army about 1939 : at Benambra in Victoria, Carrieton in South Australia and at Somerton in N.S.W. Each base was broken into two or more parts and these were measured as well as the overall lengths. The normal procedure of a measurement on each of at least two days was used, each measurement being the mean of the forward and the reverse evaluation.

Although it is generally accepted that zero corrections may be derived only from short distances, it was decided to attempt this over the bases, which are up to 9½ kms in length, deriving the corrections from the sum of the parts, and then comparing the corrected overall measurements with the taped lengths. It was thought that the bases would be short enough to have reasonably stable atmospheric conditions and yet be long enough to be somewhat representative of the geodetic distances generally measured by these instruments and that the corrections so derived would be more applicable to our work than those derived from measurements in the order of 100 - 200 metres. Also, the spread of these zero corrections would give some indication of the consistency of the MRA4 under field conditions. Results are given in Table I.

As can be seen from Table II, which summarises zero corrections at three different bases, some measure of success has been realised in Part A which, however, omits all measurements made from the northern terminal at Benambra Base. Part B, which does include these measurements is less consistent, and although it increases the spread considerably, it alters the mean zero corrections by only two to six millimetres. No reason can be given at this stage for the apparently anomalous measurements from the north terminal at Benambra as the conditions at the time appeared to be quite normal.

Comparisons of the corrected overall measurements, i.e. derived index corrections applied, with the taped lengths of the bases are somewhat disappointing, the differences being + 5.34 ppm at Somerton, + 3.43 ppm at Carrieton (Table I, A and B) and zero and - 4.41 ppm at Benambra in two similar series of measurements taken in December, 1967, and in February, 1968 (Table I, C and D). It will be noted that these results omit the northern terminal once again, Table I, E and F giving the results when it is included.

TABLE I.

MRA 4 TEST MEASUREMENTS ON THREE BASE LINES.

NOTE: In Part A measurements quoted are the mean of two to four measurements spread over two or three days. In Parts B - F measurements quoted are all means of two measurements taken on two different days. Measurements are quoted "raw", i.e. with no zero correction applied.

PART A. SOMERTON BASE LINE - AUGUST 1968.

Section of Base	Inst.Pair (1) 1011 - 1012	Inst.Pair (2) 1011 - 1017	Inst.Pair (3) 1012 - 1017
AB	1949.494	1949.493	1949.489
BC	6657.529	6657.526	6657.518
AC	8606.907	8606.900	8606.895
AB + BC	8607.023	8607.019	8607.007
AC - (AB + BC) = Zero corr'n	- 0.116 metres	- 0.119 metres	- 0.012 metres
Crystal calibration corr'n	+ 0.001	+ 0.018	+ 0.015
Corrected MRA4 distance AC	8606.792	8606.799	8606.798

Mean MRA4 distance	AC	8606.796 (spread 7 mm)
Taped length	AC	8606.842
Taped - MRA4		+ 0.046 metres or + 5.34 parts per million

PART B. CARRIETON BASE LINE - MAY 1968.

	Inst.Pair (1)	Inst.Pair (2)	Inst.Pair (3)
AB	3999.908	3999.904	3999.903
BC	2995.186	2995.194	2995.190
AC	6994.975	6994.974	6994.962
AB + BC	6995.094	6995.098	6995.093
AC - (AB + BC) = Zero corr'n	- 0.119	- 0.124	- 0.131
Crystal calibration corr'n	0.000	+ 0.005	+ 0.005
Corrected MRA4 distance AC	6994.856	6994.855	6994.836

Mean MRA4 distance	AC	6994.849 (spread 20 mm)
Taped length	AC	6994.873
Taped - MRA4		+ 0.024 metres or + 3.43 parts per million

PART C. BENAMBRA BASE LINE (NORTH TERMINAL OMITTED) - DECEMBER 1967.

Section of Base	Inst.Pair (1)	Inst.Pair (2)	Inst.Pair (3)
BC	999.505	999.511	999.493
CD	1496.684	1496.684	1496.676
BD	2496.072	2496.075	2496.054
BC + CD	2496.189	2496.195	2496.169
BD - (BC + CD) = Zero corr'n	- 0.117	- 0.120	- 0.115
Crystal calibration corr'n	+ 0.001	+ 0.002	+ 0.001
Corrected MRA4 distance BD	2495.956	2495.957	2495.940

Mean MRA4 distance BD 2495.951 (spread 17 mm)

Taped length BD 2495.951

Taped - MRA4 0.000 metres - exact agreement

PART D. BENAMBRA BASE LINE (NORTH TERMINAL OMITTED) - FEBRUARY 1968.

	Inst.Pair (1)	Inst.Pair (2)	Inst.Pair (3)
BC	999.493	999.508	999.499
CD	1496.687	1496.692	1496.692
BD	2496.068	2496.086	2496.074
BC + CD	2496.180	2496.200	2496.191
BD - (BC + CD) = Zero corr'n	- 0.112	- 0.114	- 0.117
Crystal calibration corr'n	.000	.000	.000
Corrected MRA4 distance BD	2495.956	2495.972	2495.957

Mean MRA4 distance BD 2495.962 (spread 16 mm)

Taped length BD 2495.951

Taped - MRA4 - 0.011 metres or - 4.41 parts per million

PART E. BENAMBRA BASE LINE (NORTH TERMINAL INCLUDED) - DECEMBER 1967.

$$\frac{AD+AC+BD-2AB-3BC-2CD}{4} = \text{Zero correction.}$$

	Inst.Pair (1)	Inst.Pair (2)	Inst.Pair (3)
AB	7014.341	7014.340	7014.331
AC	8013.687	8013.709	8013.692
AD	9510.313	9610.308	9510.294
BC	999.505	999.511	999.493
BD	2496.072	2496.075	2496.054
CD	1496.684	1496.684	1496.676
AD+AC+BD-2AB-3BC-2CD	- 0.493	- 0.489	- 0.453
÷4 = Zero correction	- 0.123	- 0.122	- 0.113
Crystal corr'n	+ 0.004	+ 0.007	+ 0.006
Corrected MRA4 distance AD	9510.194	9510.194	9510.187

Mean MRA4 distance AD 9510.192 (range 7 mm)

Taped length AD 9510.148

- 0.044 or - 4.63 parts per million.

PART F. BENAMBRA BASE LINE (NORTH TERMINAL INCLUDED) - FEBRUARY 1968.

	Inst.Pair (1)	Inst.Pair (2)	Inst.Pair (3)
AB	7014.334	7014.353	7014.330
AC	8013.752	8013.834	8013.803
AD	9510.313	9510.308	9510.280
BC	999.493	999.508	999.499
BD	2496.068	2496.086	2496.074
CD	1496.687	1496.692	1496.692
AD+AC+BD-2AB-3BC-2CD	- 0.388	- 0.386	- 0.384
÷4 = Zero correction	- 0.097	- 0.096	- 0.096
Crystal corr'n	.000	.000	.000
Corrected MRA4 distance AD	9510.216	9510.212	9510.184

Mean MRA4 distance AD 9510.204 (range 32 mm)

Taped length AD 9510.148

Taped - MRA4 - 0.056 or - 5.89 parts per million.

TABLE II.

SUMMARY OF ZERO CORRECTIONS - metres.

PART A.

	Inst. Pair 1011 - 1012	Inst. Pair 1011 - 1017	Inst. Pair 1012 - 1017
Carrieton Base	- 0.119	- 0.124	- 0.131
Somerton Base	- 0.116	- 0.119	- 0.112
Benambra Base (Dec.1967) (South part only)	- 0.117	- 0.129	- 0.115
Benambra Base (Feb.1968) (South part only)	- 0.112	- 0.114	- 0.117
Mean Zero Correction	- 0.116	- 0.119	- 0.119
Spread	.007	.010	.019

PART B.

	Inst. Pair 1011 - 1012	Inst. Pair 1011 - 1017	Inst. Pair 1012 - 1017
Carrieton Base	- 0.119	- 0.124	- 0.131
Somerton Base	- 0.116	- 0.119	- 0.112
Benambra Base (Dec.1967)	- 0.123	- 0.122	- 0.113
Benambra Base (Feb.1968)	- 0.097	- 0.096	- 0.096
Mean Zero Correction	- 0.114	- 0.115	- 0.113
Spread	.026	.028	.035

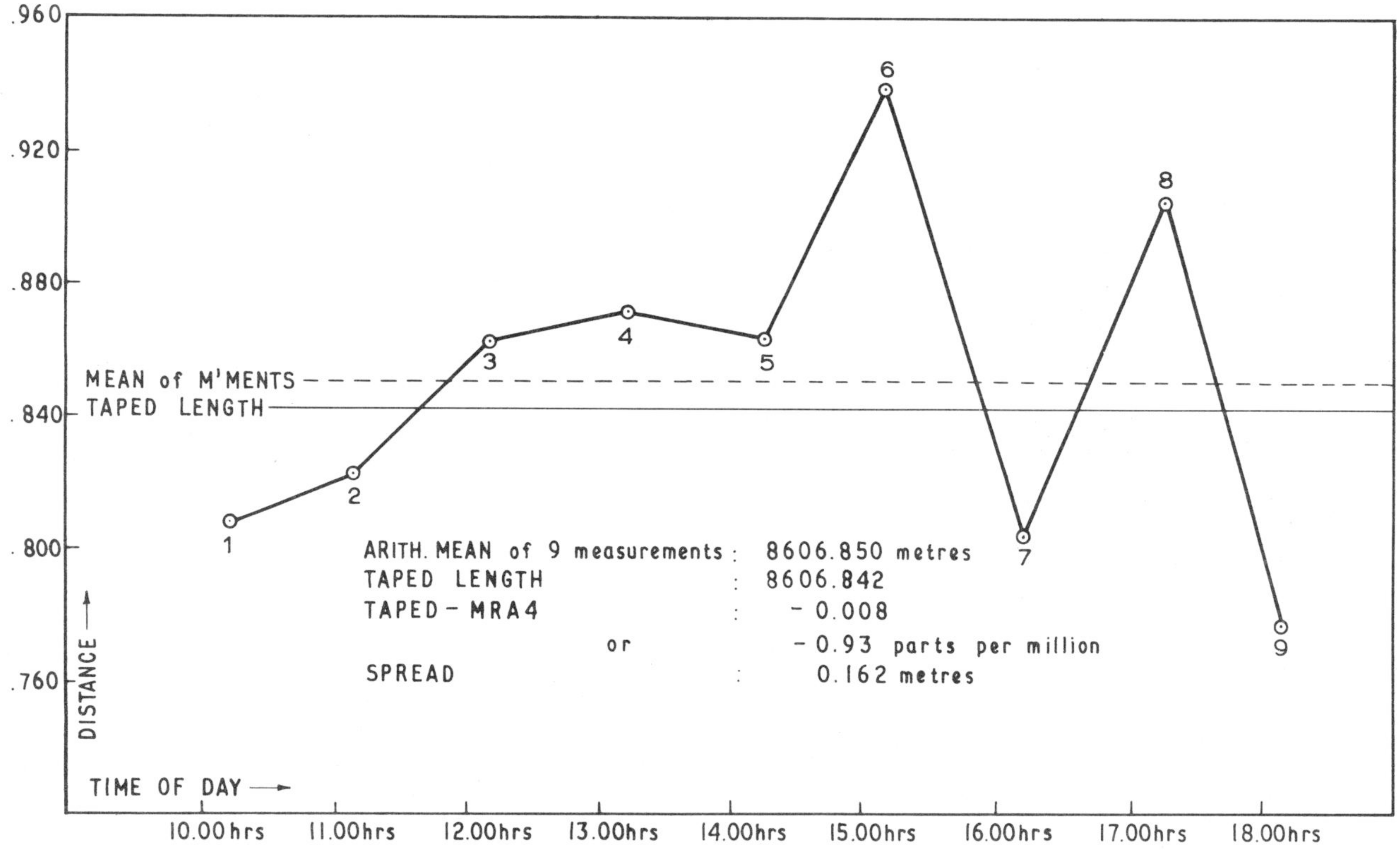


FIG. 16.1 SOMERTON BASE (NORTH TERMINAL - SOUTH TERMINAL)

NINE MEASUREMENTS OF LINE ON ONE DAY

Some improvement, no doubt, could be made using a more rigorous solution at Benambra, where the base was broken into 3 parts (Table I, E and F) instead of the approximate method used for convenience, but it is not likely that the conclusions would be altered significantly.

The zero corrections in Table II A agree substantially with those recommended by the makers and with some derived from short measurements and it can be concluded, from the spreads and the comparisons with the taped lengths, that the instruments are reliable, generally, to the limits quoted by the makers. Any desired improvement in performance will probably have to be made in determination of the atmospheric correction. It is well known to all operators of EDM equipment that this is the major source of error remaining in the measurements. It is the 'piece of wet string' on the end of an otherwise excellent measuring tape.

Some attempt has been made in the field to measure atmospheric parameters more representative of those prevailing over the whole line by moving out of the bubble of turbulent air which is so often present on hill tops, even in strong winds. It has been noticed that a move of only a few yards upwind will generally place the observer in the clean airflow, even though some elevation may be lost.

There is some evidence (Fig. 16.1) that further investigation into 'best time of day' for measuring could perhaps yield some improvement as it has done for angular measurements, at least until a better method of determining the refractive index is devised. It is hoped that simultaneous measurements on different frequencies by both optical or micro-wave techniques may eventually achieve this.

The main task of the MRA4 has been to remeasure that part of the geodetic network which runs generally east and west along the 31st parallel, from Muchea near Perth, in Western Australia to Culgoora near Narrabri in New South Wales, a distance of more than 3,000 kms. This will be used as one of the base lines in the Pageos Satellite Programme.

To date, this has almost been completed, and while it is too soon to give any specific results, some tentative comparisons are presented (Appendices I and II).

The first section (Appendix I) is from Cook to Kalgoorlie, a distance of 825 kms comprising 59 lines. It crosses the Nullarbor Plain, mainly on 20' towers, and a direct comparison of measured slope distances can be made as the same towers were used by both the MRA1's, which made the original measurements, and the MRA4's in 1967.

As could be expected, the MRA4's give a much better agreement between different measurements of the same line. The regularity of the accumulated difference seems to indicate that, although both models are measuring consistently, there could be a systematic error, probably in the MRA1 measurements. The total accumulated difference of 3.7 metres in 825 kms is, however, encouraging.

The second section (Appendix II) is from Bates, 100 miles east of Cook, to Mt. Bendemeer near Brewarrina in N.S.W. This is a distance of 1655 kms comprising 55 lines and it crosses much better going, with no towers at all. The MRA4 measurements have been compared with the values adopted in the National Adjustment and which, in this section, are not exclusively MRA1 measurements, there being some triangulation measurements included.

The agreement between different MRA4 measurements of the same line, is not quite as good as on the shorter tower-lines, being 1.6 ppm as compared with the previous average of 0.8 ppm. When compared with the adjusted lengths, the accumulated difference is similar to that in the first section in magnitude but of opposite sign, being - 3.2 metres in 1655 kms. The sum of the two sections seems to indicate an overall difference of + 0.5 metres in 2480 kms or 0.2 ppm.

It must be stressed that these are provisional results only, and that the final comparisons will not be available until the whole project is completed which will probably be during 1969.

PAPER NO. 17.

THE MRA4 TELLUROMETER IN GEODETIC SURVEYING.

S.A. Yaskowich

SUMMARY. The new Model MRA4 Tellurometer has an increased carrier frequency, narrower beam width and improved readout. It is also heavier and more expensive. Measurements in the Arctic, Nova Scotia and in Ottawa and Quebec have shown the advantages of the MRA4 over earlier models, particularly on lines with marked ground swing, very short and very long lines.

1. INTRODUCTION.

The MRA4 is the newest addition to the tellurometer series. It was designed to give a higher accuracy than that achieved to date with microwave distance measurement instruments. The improvement results mainly from use of a higher carrier frequency, improved circuitry, and higher resolution in the readout system.

The Geodetic Survey of Canada acquired 2 MRA4 tellurometers in April 1967 and obtained 2 more when they became available in the fall of 1967. The first pair was tested locally and then used on a net across Robeson Channel. This net was designed to study horizontal earth movement between Ellesmere Island and Greenland. At the end of this survey, the MRA4's were used to measure 6 lines in Nova Scotia and 4 in the Ottawa area.

During this past summer, the 4 sets were used by a Geodetic Survey party in the Quebec City area. A model 6 Geodimeter was also used and some lines were measured with both instruments.

2. THE INSTRUMENT.

Physically, the MRA4 tellurometer is slightly larger than the MRA3. In addition, a tilting head is used with the MRA4, which adds to the bulk. The combined instrument and tilting head outweigh the 30 lb MRA3 by about 15 lbs. Auxiliary equipment required such as battery, tripod, barometer and psychrometer are the same for both instruments. With regard to cost, the MRA4 is approximately two and one half times that of the MRA3.

The greatest change introduced in the MRA4 is in the carrier frequency. It is variable in the range 34,500 to 35,100 megacycles per second (Mc/s) compared with 10,000 to 10,500 Mc/s for the MRA3. Combined with a relatively large antenna reflector, the beamwidth is a narrow 2^{o} at half power points. (Compared with 9^{o} for the

MRA3). The effect of the narrow beam width is to greatly reduce ground swing and virtually eliminate it as a source of error.

Another significant change over earlier instruments is in the readout system. The numeric readout has a resolution of one millimetre (mm) and one complete cycle of the fine reading scale represents one metre. As with other tellurometers, phase errors are minimized by taking 4 phase readings per fine measurement. Also, coarse reading scales are provided for determining all digits in the distance measured. As in the MRA3, a null meter is used for indicating correct phase setting.

The circuits incorporate the newest and most reliable components and construction techniques. Removable circuit boards facilitate testing and servicing.

3. GENERAL OPERATION.

In handling the MRA4, the greater bulk and weight are readily apparent but are not considered unreasonable. Before operation, the antenna cover must be removed and the usual warm up period allowed. The narrow beam necessitates a knowledge of the direction and relative elevation of the station measured to. A compass bearing based on a map azimuth is usually adequate. An additional aid to tuning-in is a table showing corresponding master-remote frequency dial settings for each combination of tellurometers in use. On lines that are sited high over strongly reflecting surfaces it is possible to align the instruments along the reflected ray which results in an erroneous length measurement. The direct path should be identified from elevation data.

4. READOUT.

The numeric readout yields fine readings such that differences on forward and reverse positions give the fractional part of a metre in millimetres for the distance measured. On the average line, differences between forward and reverse measurements do not exceed a few millimetres. However, on very long lines, or on lines with poor signal conditions, much larger differences are observed. In such cases the null meter pointer fluctuates and causes some difficulty in taking readings.

In our operations, one set of coarse readings and 10 sets of fine readings constitute a measurement. Measurements are taken in pairs and 3 such pairs, separated by at least 4 hours and spanning at least two days, provide the minimum number of observations for a measured length.

5. GROUND SWING.

Ground swing (or swing) on a single measurement is defined here as the difference between maximum and minimum fine reading. Data available for analysis includes 48 lines measured with MRA4 tellurometers. These lines are over various types of terrain from ideally suitable to extremely poor for microwave distance measurement.

The worst terrain conditions were encountered in the Robeson Channel net. Figure 17.1 shows a sketch of this net with station elevations given in metres. The cross channel lines were sighted high over the ice and broken ice surface of Robeson Channel, the elevation of which is near sea level. The adjacent land is barren and rocky with ice, snow or water areas. A total of 29 lines with lengths of 8 to 49 Km. were measured. Measurements were made with MRA3 on 25 lines and with MRA4 on 17 lines. Eight lines had 4 or more measurements with each type of tellurometer and the data is summarized in Table I.

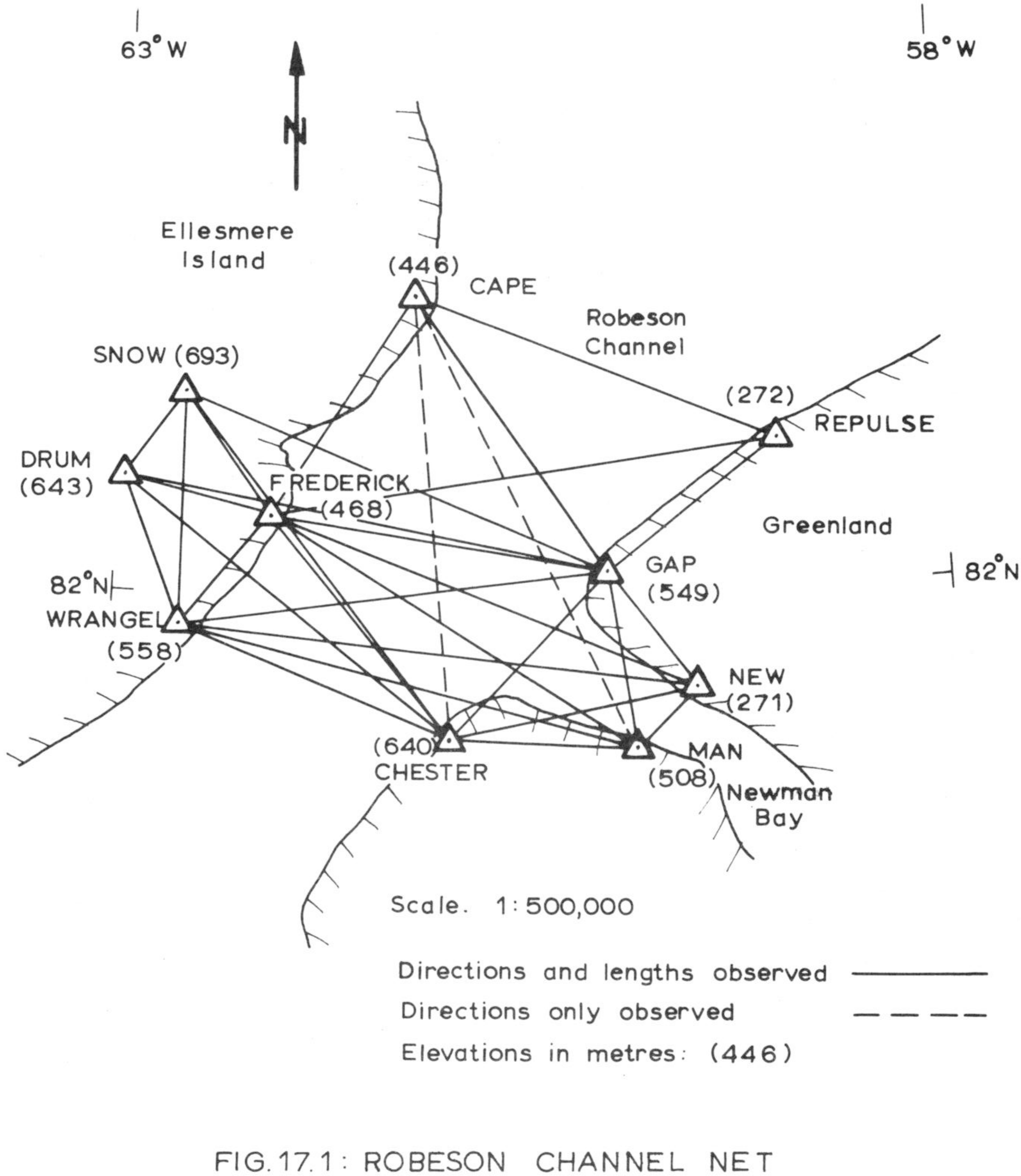


FIG. 17.1: ROBESON CHANNEL NET

TABLE I.

Comparisons, MRA3 and MRA4.

Robeson Channel Net, 1967

Line	MRA	No. Obs.	Length (Km)	Spread (cm)	Ground Swing (cm)			MRA4-MRA3(cm)
					Min	Max.	Av.	
Wrangel-Snow	3	6	22.4	23.1	16.5	107.5	60.2	-13.1
	4	8		7.3	1.8	14.0	5.8	
Snow-Drum	3	6	9.8	9.0	6.0	16.5	12.7	+ 4.7
	4	4		3.4	2.3	4.2	2.9	
Snow-Gap	3	4	43.1	23.0	75.0	133.5	106.1	- 3.3
	4	4		13.5	5.1	15.6	8.4	
Drum-Gap	3	6	46.3	32.3	33.5	166.0	95.2	-17.3
	4	5		12.9	3.4	7.3	5.3	
Chester-Frederick	3	4	26.7	26.2	28.5	45.5	39.1	+ 8.7
	4	6		11.4	1.5	7.4	4.2	
Frederick-Man	3	6	40.7	33.1	78.0	173.5	124.4	- 5.4
	4	6		12.5	5.5	8.4	6.8	
Chester-Gap	3	10	21.3	27.8	52.0	124.5	95.7	+12.2
	4	6		6.2	2.2	10.4	5.2	
New-Gap	3	8	13.6	9.1	27.0	74.5	52.6	-11.6
	4	6		3.4	4.7	8.8	6.6	
Average	3			23.0			73.2	- 3.1
"	4			8.8			5.6	

For these lines, the average swing for MRA4 measurements is 5.6 cm. and for MRA3 measurements it is 73.2 cm. The much smaller swing on MRA4 measurements is reflected in smaller average spread of measurements; 8.8 cm for MRA4 and 23.0 cm for MRA3. The largest maximum swing shown, 173.5 cm, is the second largest observed with MRA3 on this operation. On the line Chester-New ground swings on two measurements attempted with MRA3 were 4.1 and 6.7 metres. Six MRA4 measurements were made on this line for which the maximum and average swings were 7.6 cm and 5.0 cm respectively.

A summary of ground swing data for 48 lines measured with MRA4 tellurometers is given in Table II. The maximum swing, 25 cm., was observed on an 8 Km. line sighted over water in Quebec and it is interesting to note that the minimum swing on this line was 1.9 cm. Variations in swing among measurements on all lines is noted. This strongly suggests that factors other than terrain conditions, such as instrument tuning and pointing, affect the ground swing observed. The average ground swing for MRA4 measurements on the 48 lines is 4.2 cm. This is much smaller than the average 28 cm. for MRA3, and 55 cm. for MRA1 and MRA2 determined from several hundred measurements.

TABLE II.

MRA4 Ground Swing Data.

Net	No. of Lines	Av. Swing (cm)	Min.	Max.	Max. Av.*
Robeson Channel	17	5.7	1.5	16.8	8.4
Nova Scotia	6	6.5	0.9	16.0	7.5
Ottawa area	4	3.6	0.6	6.8	4.5
Quebec	21	2.4	0.4	25.0	8.3
Average	48	4.2			

* Average on line with highest average.

6. ZERO ERROR.

The antenna system, the phase measurement unit, and associated circuits are designed to give a constant zero error for each MRA4 instrument. The manufacturer provides a figure for zero error for each pair of instruments and recommends that the user check the figure by calibrating the instruments on several short, taped baselines. The zero error is of the order of + 11 cm. on a measurement.

We have conducted preliminary zero error checks on 3 slightly differing distances on a single precisely taped, 200 metre baseline. Forty-two measurements were made with one pair of instruments and gave a mean zero error of + 11.6 cm. The standard deviation for a single measurement is ±1.6cm and the maximum difference from the mean for a measurement is 3.2 cm. The standard deviation of the mean is ±0.3 cm. The zero error given by the manufacturer for this pair of instruments is + 10.4 cm., which is 1.2 cm smaller than the value we obtained.

The zero error observed depends on such factors as characteristics of the base, the number of fine readings taken per measurement, the carrier frequency range used, and the heights of instruments.. With regard to the latter, 6 measurements made with instrument heights of 1.2 metres gave an average zero error of +13.3 cm., and 6 measurements made at the same time with instrument heights of 1.8 metres gave an average zero error of +11.8 cm. A possible explanation for the difference is that the ground swing cycle is not as completely developed on the measurements with the instruments set low. These measurements were included in the measurements analyzed above.

In the zero error measurements made to date, no cyclic variation has been detected. Further zero error checks are planned.

7. MODULATION FREQUENCY.

The master crystal which governs the accuracy of measurement has a frequency of the order of 7.5 Mc/s and is housed with the coarse frequency crystals in a thermostatically controlled oven. On the two instruments in use all last season, crystal frequencies were calibrated before and after the field season as well as once a month during the season, in accordance with our normal practice. The fre-

frequency drift was one to two cycles/second/month and compares favourably with an average drift of one cycle/second/month for our MRA3 crystal frequencies. Frequency drifts of such small magnitude result in negligible frequency error when appropriate corrections based on calibration data are applied.

8. INDEX OF REFRACTION.

Errors in determination of index of refraction for MRA4 measurements have the same effects as they do on measurements made with other microwave instruments. On MRA4 measurement, this is the only major source of error present and the accuracy attained depends on how effectively meteorological errors are controlled.

To date we have not made an intensive study of the problem, but in the future, studies will be made of methods for reducing meteorological errors. Such methods include measuring a line in two sections; elevating the psychrometers at least 10 metres above ground; observing meteorological data at points along the line; doing a more comprehensive analysis of meteorological data; and restricting observations to periods of near ideal meteorological conditions.

9. COMPARISONS BETWEEN MRA4 AND LASER GEODOLITE.

Two lines in the Ottawa area were measured with MRA4 tellurometers and the laser geodolite. The following results were obtained with MRA4's:

TABLE III.

Length Km.	No. of Days	No. of Measures	M.s.e of mean		Difference Geodolite - MRA4	
			cm	ppm	cm	ppm
4.4	3	30	0.5	1.1	+1.3	2.9
16.0	2	6	1.0	0.6	+0.5	0.2

As stated previously some lines in Quebec were measured during the summer of 1968 with both MRA4 tellurometer and model 6 geodimeter. Table IV, Comparison between MRA4 and geodimeter 6 Measurements, shows the results obtained on 14 lines measured by both types of instruments.

TABLE IV.

Comparison Between Tellurometer MRA4 and Geodimeter 6 Measurements.

Lines shown in the following table were measured in Quebec in 1968 using both MRA4 tellurometer and Model 6 geodimeter. The tellurometer measurement on each line is the mean of a minimum of 6 measurements taken over a two day period. A geodimeter line measurement consists of 6 observations on each of the 3 frequencies and the mean of the 18 observations constitutes the measured length.

Line	Tellurometer MRA4 (m)	Geodimeter 6 cm.	Minus MRA4 ppm
Orleans - Marquis	3 849.567	-0.9	2.3
Orleans - Monte Ste Anne	9 667.380	-6.0	6.2
St. Joachim-Monte Ste Anne	7 590.206	-3.4	4.5
Marquis - Monte Ste Anne	10 434.665	-2.0	1.9
Lebel - Anse A Giles	9 023.136	-1.9	2.1
Lebel - Oie	10 566.610	-3.2	3.0
Anse A Giles - Oie	7 881.659	-0.3	0.4
St. Pierre - Rocher Noir	6 720.879	-2.6	3.9
Lauzon - Martel	8 144.759	-1.5	1.8
Lauzon - Dominicaine	7 195.059	+7.2	10.0
Lauzon - Tour	7 978.135	+3.9	4.9
Morin - Tour	14 812.357	+6.0	4.0
Dominicaine - Tour	8 537.784	+0.1	0.1
Delisle - Morin	5 400.548	-1.7	3.2

10. CONCLUSION.

The small ground swing on MRA4 measurements represents a great improvement over earlier tellurometer models and virtually eliminates this source of error. Refinements in data read-out further reduce reading and zero error.

The limited comparisons with geodolite measurements indicate that high accuracy can be achieved on shorter lines. It is anticipated that reliable results can also be expected on longer lines, including those over 30 kilometres which are not normally measureable by electro-optical instruments, but on such lines every precaution must be taken to minimize meteorological errors.

The instruments are well suited for short line measurements and can be used for routine survey work but their high cost is a restrictive factor. They are most useful for accurate measurements on lines sighted over highly reflective surfaces and in surveys where a higher accuracy than that normally obtained with other microwave instruments is required.

PAPER NO. 18.

THE MEASUREMENT OF THE SWEDISH-NORWEGIAN SECTION OF THE TROMSÖ - CATANIA SATELLITE BASE LINE.

I.R. Brook.

SUMMARY. During the 1967 and 1968 field seasons the Norwegian and Swedish sections of the Tromsö - Catania baseline were measured using MRA4 Tellurometer and Model 4 Geodimeters with lasers. The field operations, assisted by helicopter in the northern sector, were carried out with care and computations show that a very satisfactory accuracy was achieved. The field performance of the instruments was satisfactory.

1. INTRODUCTION.

During the 1967 field season a field party from Rikets Allmänna Kartverk measured the 56 sides which comprise the Norwegian and Swedish section of the Tromsö - Catania satellite base-line. Third generation, high precision EDM equipment was used for the measurements, namely two model 4 Tellurometers serial numbers 1003 and 1004 and a modified model 4D Geodimeter in which the standard light source was replaced by a 2 milliwatt helium-neon gas laser.

A number of check measurements were made during the 1968 season. The greater number of these measurements were made using a laser geodimeter although a number of distances previously measured with model 4 Tellurometers, principally water lines, were remeasured with the same equipment.

The route of the traverse through Sweden is shown on the sketch map of the Swedish first-order triangulation (Fig. 18.1). As can be seen, the distance measurements have been carried out as a single continuous traverse which follows the backbone chain of the national first-order network. The line of the traverse was, to a certain extent, determined by the desire to avoid expensive tower building or to make use of existing towers. Between the final Swedish station in the north and Tromsö, the traverse follows a part of the Norwegian first-order trilateration network. In the middle and southern part of the traverse, which runs through forested areas, special wooden distance measuring towers were built. For this construction only a single tower is required, the observer's platform being an integral part of the instrument tower. The average tower height in this section of the traverse is 15 metres.

From station 223 northwards the traverse skirts the eastern edge of the mountainous areas which make up the western part of Sweden at these latitudes. The average height of the stations increases northwards to a maximum elevation of over 1600

metres in the middle of the Norwegian mountain section, thereafter decreasing rapidly to 100 m above sea level at the terminal camera station in Tromsö.

As transportation presents considerable problems in the highland areas, two Hughes 300A helicopters were chartered for an eight week period for transporting observers and equipment. This type of helicopter is capable of transporting the full equipment with pilot and a surveyor. The operating range with this load and full fuel tanks is approximately 250 kms. To speed up the observations the helicopter pilots were trained to book the observations.

For the remainder of the traverse, back-packing was necessary. The field party which previously had consisted of a head of party and a surveyor was, therefore, increased by an additional surveyor and four labourers.

Progress in this part of the traverse was slower due largely to adverse weather conditions. The final connection to Denmark was completed on 25th October. This connection was made in co-operation with the Danish Geodetic Institute. The total length of the traverse is 1844338 metres.

2. THE FIELD WORK.

At the planning stage it was decided that all sides in the traverse would be measured using the tellurometers and that check measurements at frequent intervals would be made using the laser geodimeter if weather conditions permitted.

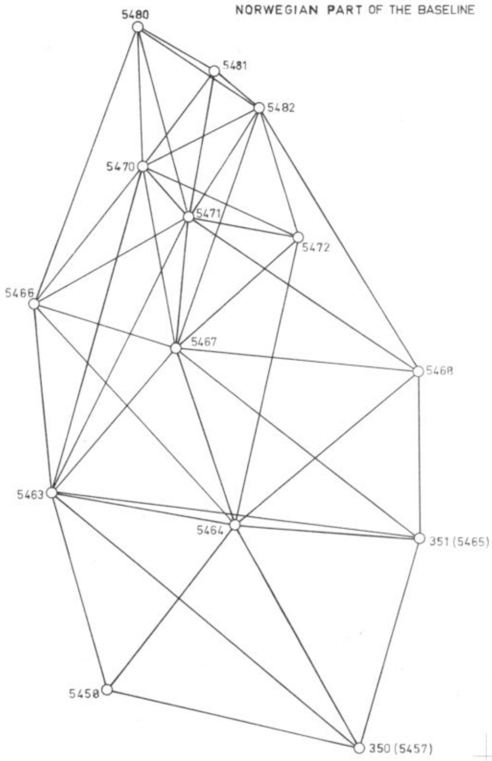
Field work began at the beginning of July at station 265 and during the first month the rate of progress northwards was approximately 130 kms per day, i.e. four tellurometer measured distances. Progress was considerably slower in the Norwegian mountains where very poor weather grounded the helicopters. The connection to the Nordlyse camera station in Tromsö was measured at the end of July. It had been planned to measure all tellurometer sides three times but the continuing poor weather in Norway and the desire to use the helicopters as far south as possible during the charter period made it impossible to measure the sides between station 350 and 5481 more than once. The measurements were, however, made under good tellurometer conditions: heavy cloud, strong wind and an average dry bulb temperature of approximately $5^{o}C$. The distances had also been measured previously by the Geographical Survey of Norway using MRA 3 tellurometers.

The helicopters were used, thereafter, as far south as station 136. During the helicopter period, which extended from 5th July to 31st August, 41 tellurometer distances were measured. As all sides except the five most northerly were measured three times, this represents a total measured tellurometer distance of 4328791 metres or 87000 m per working day.

In all, 50 distances were measured using the tellurometers. In 1967, seven of these distances were checked using the laser geodimeters and six of them had previously been measured using earlier types of geodimeters. In the southern part of the traverse five sides which had previously been measured to first-order standards using standard Model 4D geodimeters have been incorporated in the traverse. These sides were not remeasured with the tellurometers.

The connection to Denmark was measured by tellurometer. A geodimeter check of the distance was planned in 1967 but despite a three week wait the weather was never sufficiently good to make the measurement possible. This side which is 47000 m long and the longest in the traverse, was measured with the laser geodimeter during the 1968 season. Despite the length of the line, good signal strength was obtained even in half-light conditions. Thirty-two prisms were used for this measurement. The side

THE SWEDISH PART OF THE TROMSÖ – CATANIA BASE LINE



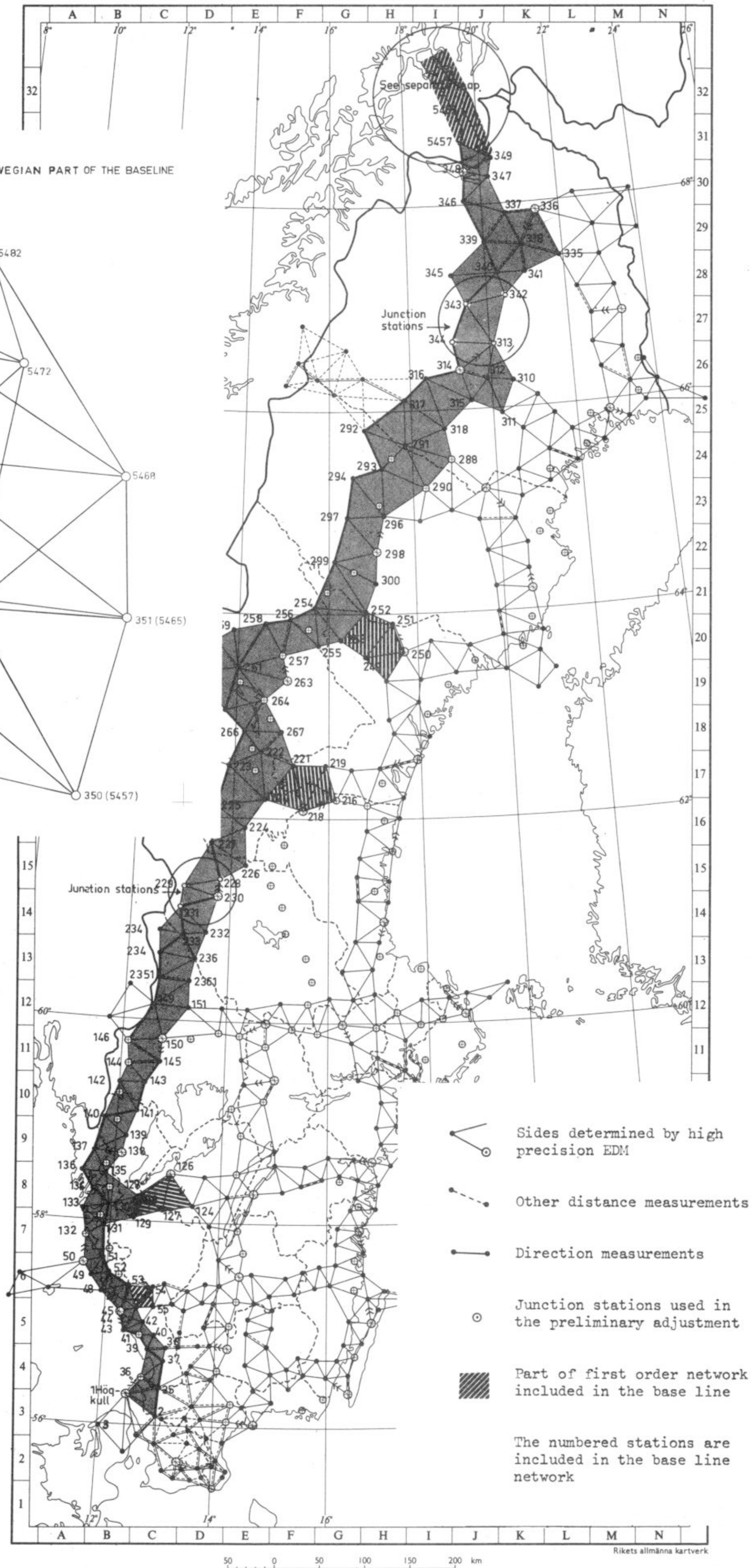


Figure 18. 1

was also broken down into two sections of 25000 and 22000 m respectively and measured with the geodimeter equipment. The degree of agreement between the measurements of the whole distance, the computed value from the two sections and the tellurometer values, is high.

Ancilliary equipment. Temperature observations were made using the large type of Assmann clockwork-driven psychrometers. The large type of Paulin aneroid was used for pressure measurements. The aneroids were calibrated against a mercury standard at the beginning and end of the field season. The field aneroids were checked each morning and evening against two station aneroids which were kept in the field quarters. During the 1968 season Baromec equipment was used, because of its high degree of stability and accuracy, for the daily checks of the field aneroids. This, in effect, resulted in a speed-up of the computational procedure since absolute corrections which earlier had been computed after laboratory comparisons between the mercury standard and the station aneroids were now obtained from the daily field comparisons.

High quality thermometers with an accuracy of 0.1°C were used in the psychrometers. The psychrometers were checked against each other on several occasions during the course of the field work. The speed of the fan motor is easily checked in the field and such checks were carried out frequently.

No frequency counter was available in the field in 1967. The tellurometer frequencies were, therefore, checked at the beginning, in the middle and at the end of the field period. For the tellurometers, light-weight 32 AH batteries were used as the power source.

An AGA frequency meter, specially produced for use with geodimeter equipment, was used in the field to check the geodimeter frequencies at regular intervals. During the 1968 field season the field party was supplied with a portable counter. A light weight AGA portable two-stroke motor generator was used with the geodimeter. This has since been replaced by a Honda four-stroke motor-generator, which has proved to be extremely reliable.

Geodimeter measurements. Operation of the laser instrument is, from an operator's point of view, basically the same as a standard 4D instrument.

Normal long delay-line measuring procedure was used for all measurements. On each frequency both high and low delay-line observations were made where this was possible. Wet and dry bulb temperatures and pressure were recorded at both terminals.

To facilitate pointing of the instrument, since there is no search-light on the modified geodimeter, a 75 W searchlight was used at the reflector station. Portable 5 W radio equipment was used for communication between the geodimeter station and reflector station. The desirability of having radio contact when long distances are to be measured, particularly under daylight conditions, was clearly demonstrated in the south of Sweden where distances of the order of 20 000 metres were measured in conditions of sunlight and light haze. Under such conditions the reflected signal is often not visible in the geodimeter pointing telescope and pointing must be carried out with the help of reports from the mirror attendant. Of the sides measured with the laser geodimeters only one was double-measured.

The geodimeter used during the 1967 field season was replaced prior to the beginning of the 1968 season by a similar, but improved instrument. Although both instruments are essentially identical it seems that the laser in the second instrument was more powerful than the laser in the earlier instrument and that the phototube in the 1968 instrument was more sensitive than the equivalent tube in the 1967

instrument. As both lasers are nominally of the same power it would seem that the laser used in the first instrument was not performing effectively. Power losses due to the modulation system and the transmitting optics are of the order of 50% in both instruments.

Tellurometer measurements. Prior to the start of the field season the MRA 4 tellurometers were tested in the field and an observing programme was established. The results of these tests have been reported in an earlier paper.

Four MF readings: MF + forward, MF - forward, MF - reverse, MF + reverse were observed on five frequencies. The Remote cavity dial settings are 4, 6, 8, 10 and 12. The test measurements indicated that no appreciable increase in accuracy was attained by increasing the number of settings. It should be stated that the same dial settings are used when the instruments are calibrated.

The only exception to this observing procedure was the long water line connecting Sweden and Denmark where 10 fine readings were taken. The measuring procedure was as follows:

	Instrument 1003 as Master	Instrument 1004 as Remote
1.	Cavity setting : 3.5 Low	Cavity setting: 4 High
2.	Coarse Readings	
3.	Meteorological observations	Meteorological observations
4.	Fine Readings	
5.	Meteorological observations	Meteorological observations
6.	Switch to Remote position: Cavity dial 4 High	Switch to Master position: Cavity dial 3.5 Low
7.		Fine Readings
8.	Meteorological observations	Meteorological observations
9.		Coarse Readings

Coarse readings were only observed for two of the three measurements. One coarse reading, when this is made with both instruments and when the coarse distance is broken out at the time of the measurement and checked before switching off the instruments is, in fact, normally sufficient. But some lines in the north of Sweden, when the variations in refractive index between the three days were large, gave M2 - M1 values, which varied by a meter. In some cases this could lead to confusion.

All distances were measured three times. With the exception of a few sides in the southern part of the traverse, these three measurements were carried out on three separate days. Normal Swedish first-order practice calls for at least a one day interval between each of the three measurements. Due to the need of maintaining a rapid rate of progress and the desire to utilize the helicopters to the full this was seldom possible. The measuring practice mentioned above is based on the desire to reduce the possibility that the observed refractive indices will not be representative, by making the three measurements under different weather conditions. However, weather conditions can change very rapidly, particularly in the mountain areas of the country, and accuracy does not appear to have suffered in any way by this concentration of the measuring programme.

When a side was measured three times on two days the two measurements which were made on a single day were made with a considerable time interval between them such that a measurable change in the refractive index could take place.

Meteorological observations. At all stations where measurements were made from tripods, temperature observations were taken 2-3 m above the ground with a view to reducing ground radiation effects,as far as possible, and to try to compensate for the different inertia characteristics of the wet and dry bulb thermometers.

The psychrometers were protected from direct sunlight and, as far as possible, shaded from heat reflections from nearby illuminated objects. In towers, the psychrometers were hung in the shade outside the observer's platform.

Poor aspiration of the wet bulb can be a source of serious error. It is therefore of great importance that the motor is sufficiently powerful to ensure that the velocity of the air past the bulbs is not less than 2 m/sec. Investigations have shown that the velocity of the air must be greater than 2 m/sec. but that velocities up to 5 m/sec. are in no way detrimental to accuracy.

Standard observing procedure is to tilt and point the psychrometer slightly against the wind thereby increasing the air velocity past the thermometer bulbs and ensuring that ventilation is always adequate. When wetting the wet bulb sleeve, care was taken to prevent water drops forming a bridge between the wet bulb and the shield. Observations were always continued until equilibrium was reached.

The aneroids were also placed in the shade and protected, as far as possible, from undesirable external effects: i.e., turbulence effects resulting from gusty wind conditions. No mid-line observations were made.

Zero error corrections. Zero error corrections for both the tellurometers and geodimeters were determined before the beginning of the field work. In addition a further zero error determination was carried out before the connection to Denmark was made. The results of the tellurometer zero error measurements are given in Table I.

TABLE I.

Tellurometer Zero Errors.

Date	Instr 1003	Instr 1004
May 1967	- 109 mm	- 110 mm
October 1967	- 111 mm	- 114 mm
April 1968	- 107 mm	- 116 mm

3. INSTRUMENT PERFORMANCE.

The impressions gained during the course of the field tests which were carried out before field-work in connection with the traverse and which have been presented elsewhere, have been confirmed.

Both instruments have functioned satisfactorily with no need for other than minor repairs or adjustments.

The only fault encountered with the tellurometers was a jamming cavity-change dial which was corrected in the field.

Certain problems were encountered with the photo-tube unit and the K.D.P. modulator when using the original geodimeter under cold, windy, damp conditions. Under such conditions sensitivity was greatly reduced and changes of phase could occur. In the instrument used during the 1968 season these faults have been eliminated.

Excellent daylight range for the geodimeter and greatly reduced swing tendencies with the tellurometers are the two improvements over previous models which have made the most positive impressions.

The already excellent range of the laser geodimeter was even more impressive when the second instrument was put into use. Daylight range in conditions of haze and bright sunshine was of the order of 20 000 - 25 000 m. Under such conditions the principal problem is still that of locating the prisms.

In the north of Sweden during June and July, the nights are never dark but, nevertheless, under half-light conditions several sides of between 30 000 and 40 000 m were measured. The longest distance measured was 47 000 m; and it can be mentioned that a fully measurable signal was obtained on a 20 000 m line using a single prism. A night-time range of between 50 000 - 60 000 m would appear to be achievable with this instrument under good measuring conditions. In high precision geodetic work it is hardly advisable to measure such long distances due to the difficulty in determining the refractive index and also the problems associated with the k value. Night range is, thus, mainly of academic interest. Of greater interest is the good daylight range and performance; up to distances of 30 000 m.

In an earlier report, mention was made of a tendency towards fading when using the tellurometers on water lines. This tendency has been observed on several water lines measured during the 1968 season. On some lines the severity of the fading was such that contact between the instruments was broken. Under such conditions the needle of the undamped null indicator can fluctuate wildly and make a visual integration extremely difficult. On one and the same line the tendency towards fading could vary from slight to severe when the measurements were repeated on different days with varying wind and wave conditions. Work in Canada with MRA 4's on highly reflective lines has confirmed our experience that, under poor visibility conditions, it is possible to point an instrument to a strong reflected signal instead of to the direct signal.

A number of typical tellurometer swing curves are given in Fig. 18.2. Antenna swing is of the order of 15 mm.

4. REDUCTION OF THE MEASURED DISTANCES.

The standard, recommended Essen and Froome formula has been used for computation of refractive index for the tellurometer measurements. The refractive index is computed separately for each station and not from the mean of the temperature and pressure readings at the two stations.

For the geodimeter measurements the following formula has been used

$$1 + \frac{0.000107925p}{273.2 + t} - \frac{1.5026\ e\ 10^{-5}}{273.2 + t}$$

p = pressure in mm Hg
t = temp in ^{o}C
e = vapour pressure in mm Hg

-241-

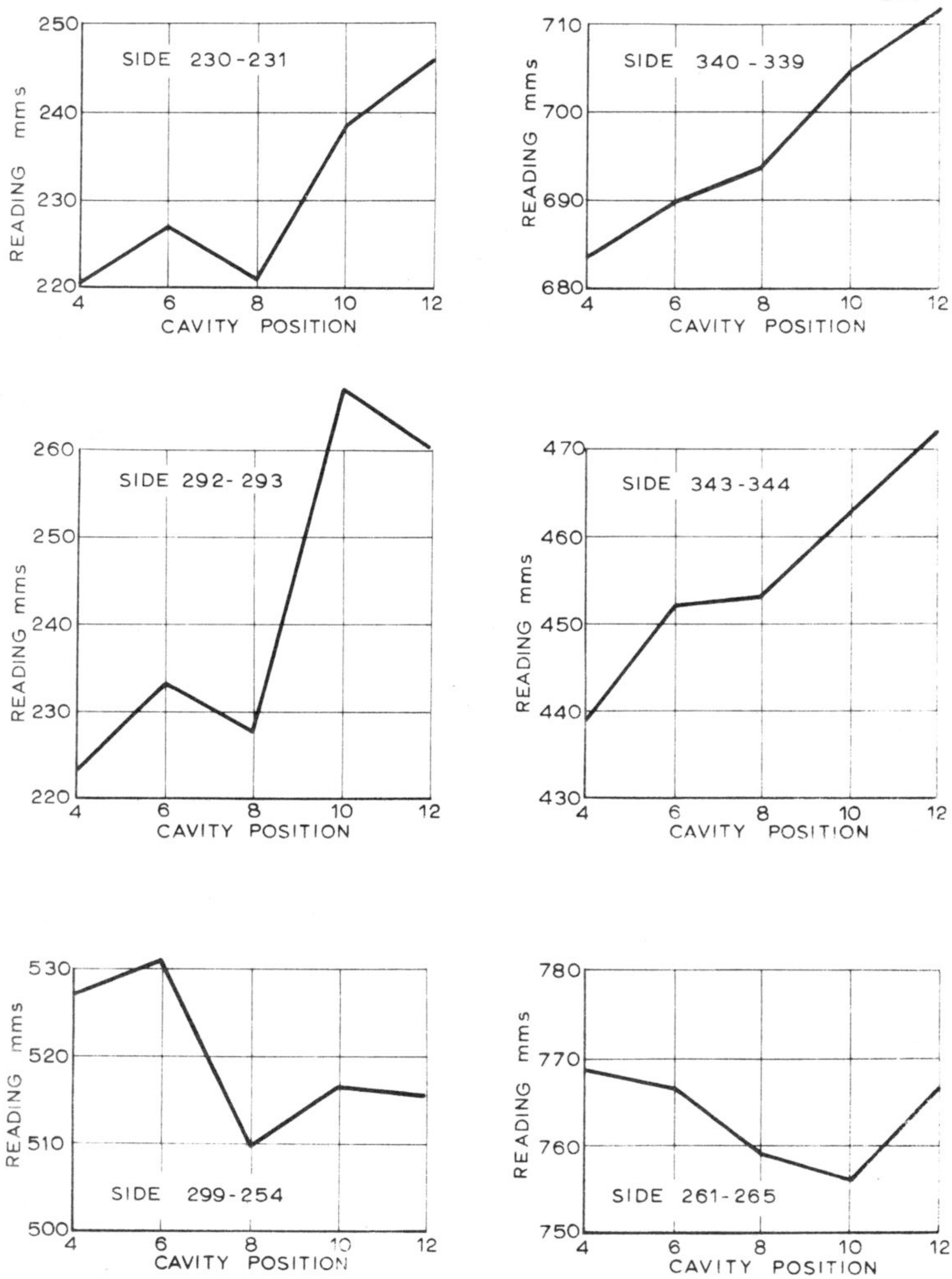


FIGURE 18.2 TYPICAL TELLUROMETER MRA4 SWING CURVES

The wavelength of the laser used in the modified instrument is 6328A.

Standard geometrical corrections have been used to reduce the measurements to the reference ellipsoid. In addition, first and second velocity corrections i.e., the correction for path curvature $\left(-k^2 . \frac{D^3}{24R^2} \right)$ and the correction for the dip of the ray path into layers of higher refractive index $\left(- (k-k^2) . \frac{D^3}{12R^2} \right)$ have also been applied.

The values adopted for k are 0.25 for microwaves which is equivalent to a $\frac{dn}{dh}$ value of - 0.039 . 10^{-6}/metre and for light 0.20 which is equivalent to a $\frac{dn}{dh}$ value of - 0.031 . 10^{-6}/m. According to Höpcke the k value 0.20 is a representative value during the day and on overcast nights.

For distances of up to 30 kms the choice of k value is in no way critical. Checks of the k value were carried out by measuring vertical angles at the geodimeter station to the prisms.

The geoid profile along the traverse was completed during the 1968 field season. Geoid height corrections have been applied to all measured distances.

The ranges of the values of the three tellurometer measurements which comprise a complete determination expressed as a proportion of the measured distance are given in tabular form in Table II. Under circumstances of varying atmospheric conditions such as were often experienced during the measurement of the traverse, these range values can, to a certain extent, be considered as an indication of the accuracy of the determination of the refractive index.

TABLE II.

Tellurometer measurements - The spread between outer measured values expressed as a proportion of measured distance.

spread/distance	total number of sides
1/200 000 - 1/250 000	5
1/250 000 - 1/350 000	9
1/350 000 - 1/500 000	9
Under 1/500 000	22

The average difference between the outer values of the three measurements expressed as a proportion of the distance is 1/400 000. The maximum difference in the whole traverse is 1/160 000 which is the only measurement with a proportional difference greater than 1 200 000. Normally, measurements with a proportional difference between the outer values of greater than 1/200 000 of the distance are remeasured. Bad weather prevented remeasurement of the side quoted above.

Comparisons between the geodimeter and tellurometer determinations of those sides which were measured with both instrument types are given in Table III. The average difference between the determinations is 1/812 000 with a maximum difference of 1/350 000.

The comparisons between measurements made by the Geographical Survey Office of Norway with MRA 3 tellurometers and the measurements made by the Swedish team are given in Table III in which is also given comparisons between measurements carried out with MRA 4 tellurometers both in 1967 and 1968.

TABLE III.

Comparison of measurements - metres.

A. Comparison of distances measured with MRA 4 and MRA 3 tellurometers.

MRA 4 metres	No.of meas.	MRA 3 m.	No.of meas.	Diff.MRA 4 - MRA 3 m.	proportional difference
32814.095	1	32814.145	5	-.050	1/660 000
45360.796	1	45360.669	2	+.127	1/360 000
22912.284	1	22912.336	2	-.052	1/440 000
26234.826	1	26234.779	3	+.047	1/560 000

B. Comparison of distances measured with MRA 4 tellurometers and laser geodimeters

side	geodimeter m.	tellurometer m.	Diff. g-t m.	proportional difference
316-317	32216.862	32216.896	-0.034	1/950 000
258-256	25629.250	25629.242	+0.008	1/3200 000
228-230	21908.348	21908.295	+0.053	1/410 000
2351-235	20695.488	20695.532	-0.044	1/470 000
2351-149	29832.187	29832.248	-0.061	1/490 000
143-145	28364.849	28364.921	-0.072	1/400 000
136-134	25298.308	25298.298	+0.010	1/2500 000
130-134	27000.059	27000.062	-0.003	1/9000 000
53-52	26371.977	26371.908	+0.069	1/375 000
51-52	15382.271	15382.288	-0.017	1/900 000
43-45	19998.442	19998.417	+0.025	1/800 000
43-41	18478.512	18478.508	+0.004	1/4600 000
36-37	28274.692	28274.705	-0.013	1/2200 000
36-1	23517.344	23517.277	+0.067	1/350 000
1-3	47039.837	47039.810	+0.027	1/1742 000

C. Comparison of distances measured with MRA 4 tellurometers 1967 and 1968

side	MRA 4 1967 m.	MRA 4 1968 m.	Diff. m.	proportional difference
1-3	47039.770	47039.849	+.079	1/595 000
43-45	19998.400	19998.434	+.034	1/588 000
36-37	28274.707	28274.703	-.004	∞

5. PRELIMINARY COMPUTATION OF THE STANDARD ERROR IN THE GEODESIC BETWEEN HOGKULL AND TROMSO.

The map, Fig. 18.1, illustrates the scope of the adjustment. As can be seen, the computations were carried out in three separate zones which were then, in the final stage of the adjustment, connected in accordance with the RETRIG principles.

In addition to the data from the traverse, all available first-order angular and distance measurements and Laplace observations which fall within the part of the net as shown on the map where the stations are numbered were included in the adjustment.

The 'standard error à priori' for an observed direction was 1^{cc} and for distances measured with tellurometer MRA-4 and first-order geodimeter measurements $\{10 + s \cdot 10^{-6}\}$ mm and for measurements with MRA 101 $\{20 + s \cdot 10^{-6}\}$ mm.

Station 1-Högkull was the only fixed point and Tromsö - Nordlyse was chosen as the final junction station. Using Cholesky's elimination procedure the following triangular matrix was obtained for Nordlyse (the units used are centesimal seconds and decimetres).

0.21632	0.04921	0.49681
	0.09111	-1.36847
+ 5.713	-15.020	709.774
$x = Md\phi$	$y = N\cos\phi d\lambda$	$\sum pvv$

$\mu^2 = 709.774 : 289 = 2.456$

$\mu = 1.567$

The inverse matrix for Nordlyse is :

27.6043	- 27.4034	Q_{xx}	Q_{xy}
-27.4034	120.4594	Q_{xy}	Q_{yy}

The standard error in the geodesic between Högkull - Nordlyse

$ds = -\cos\alpha\, dx - \sin\alpha\, dy$

Tienstra's formula gives:

$Q_s = -\cos\alpha\, Q_x - \sin\alpha\, Q_y$

$Q_{ss} = \cos\alpha\, Q_{xx} + 2\sin\alpha\cos\alpha\, Q_{xy} + \sin^2\alpha\, Q_{yy}$

α_{Nor} - Hög ~ 216.9 $\cos\alpha = -0.96497$

$\sin\alpha = -0.26235$

$Q_{ss} = 0.93117 \cdot 27.6043 - 0.50632 \cdot 27.4034 + 0.06883 \cdot 120.4594$

$Q_{ss} = 20.121$ $m_s = \pm 7.0$ decimeters

The error ellipse is computed from $\begin{vmatrix} Q_{xx} - \lambda & Q_{xy} \\ Q_{xy} & Q_{yy} - \lambda \end{vmatrix} = 0$

which gives $\lambda_1 = 20.120 \quad \lambda_2 = 127.943$

The axes of the ellipse are $\mu\sqrt{\lambda_1} = 7.03 \quad \mu\sqrt{\lambda_2} = 17.73$

and the azimuth of the major axis is $116^g.0$

The distance Nordlyse - Högkull is approximately 1 524 kms

In Table IV the size of the corrections in relation to the 'standard deviation à prior' are analysed. The largest correction to a measured distance in relation to the 'standard error à priori' is $\frac{10.2}{4.8}$ for side 261 - 260.

The largest corrections to measured directions in relation to 'standard error à priori' are for

directions 315 - 314 : $\frac{3.76}{1.00}$ and 296 - 290 : $\frac{3.03}{1.00}$

TABLE IV.

Corrections to measured distances in relation to the 'standard error à priori'

	Mean Side length -kilometres	$\leq$ 1x	1 - 1.5x	1.5 - 2x	$\geq$ 2x	Mean adjustment corrn-cms
Zone I	26.5	28	5	1	1	2.9
Zone II	38.3	19	6	1	1	3.8
Zone III	36.6	58	8	2	1	4.1
Total		105	19	4	3	

The standard error obtained in these preliminary calculations gives a good estimate of the accuracy in the distance determination between Högkull and Tromsö as far as the influence of random errors in angular measurements, distance measurements, height determinations and Laplace azimuth determinations are concerned.

The estimate will be practically the same in a computation where the measurements have been reduced for geoidal heights. However, errors of a systematic character will considerably decrease the real accuracy. Amongst such sources of error the following can be mentioned:

1. error in the velocity of light.
2. systematic errors in frequency control
3. errors in the determination of the geoid profile and particularly in the determination of the geoidal height of the end points.

In view of this, the estimated accuracy from random error sources must be considered very satisfactory. The adjustment was carried out by the Head of the Computing Section, Mr. Ilmar Ussisoo.

DISCUSSION: PAPERS 14 - 18.

Chairman: Mr. B.P. Lambert

ROBINSON:	Tellurometer Zero Error
LEHR:	Optimum Signal (Cionini, Mezzani)
BOBROFF:	MRA4 in Australia
GALE:	MRA4 in Canada (Yaskowich)
MATHER:	Measurements in Sweden (Brook)

CHAIRMAN: Since Robinson's investigations have shown a cyclic component of zero error, do measurements taken at half the cyclic interval apart eliminate the cyclic error?

ROBINSON: In theory the cyclic effect should be eliminated by this procedure. With the MRA101 the cyclic error is small compared with standard deviation of measurement, so that the procedure has not been tested in my experiments.

MILLER: I have carried out some investigations with the tellurometer MRA2. The cyclic error was more pronounced and grouping observations at the half-cycle interval apart gave a significantly smaller standard deviation.

SIMMONS: What was the effect of tilting the tellurometers? In view of the plumbing procedure surely this caused an error.

ROBINSON: The two instruments were tilted through the same angle in opposite directions so that the tilt error cancelled.

CLEGG: In the paper by Cionini and Mezzani there is no mention of variation of the time of modulation and this is something which could have an effect on band width and power required.

McQUISTAN: The paper seems to indicate that the optimum system would be a swept frequency system.

LEHR: This might be the case, theoretically, but in practice the technological difficulties of setting up such a system might outweigh the saving in power.

CHAIRMAN: Have comparisons been made between tellurometer MRA4 and geodimeter measurements?

BOBROFF: No direct comparisons have been made. However three bases have been measured on different occasions. Geodimeter - tape differences on the three bases were +3.62, -0.70 and -1.58 p.p.m. whereas tellurometer - tape differences were +5.34, +3.43, and -4.41 p.p.m.

BENNETT: The difference between the MRA4 and MRA1 measurements is systematic and about 4.5 p.p.m. and the published loop closures are 2 p.p.m.

BOBROFF: MRA1 measurements included a zero correction obtained from comparison with geodimeter measurements. The difference between the two sets of tellurometer measurements is most likely due to this index correction.

CHAIRMAN: The re-observation of some of the distances by geodimeter will be undertaken in the future as a further test of the accuracy.

GALE: In Canada, triangulation sides have been measured both by geodimeter and tellurometer. Good agreement has been obtained near the base lines but in the middle of a chain discrepancies are up to $^{1}/30\ 000$.

BOBROFF: Australian experience confirms this.

BROUGHTON: Do you use any criterion in the field to decide whether meteorological conditions are satisfactory for observation of distances to first order standards?

BOBROFF: 'Met.' observations are taken at both ends of the line and both refractive indices calculated in the field. If these agree within empirical limits, based on experience, the distance measurements are taken.

LYONS: What is the best time of day for observing in Australia and in Canada?

BOBROFF: We have insufficient information on this subject but there are indications that the best times for distance measurement are the same as for vertical angle measurement: 10 a.m. - 3 p.m.

GALE: Most distance measurement in Canada takes place in this period, but from considerations of convenience rather than precision.

LYONS: In the MRA 4 measurements on the east-west geodetic traverse,were any measurements rejected and if so, what was the rejection criterion?

BOBROFF: No measurements have been rejected.

LYONS: Is the Geodetic Survey of Canada using error ellipse techniques to analyse the differences between the MRA 3 and MRA 4 measurements in the Robeson Channel net.

GALE: The net included angle measurements as well and it will be some time before the data can be fully analysed.

FRYER: What accuracy is National Mapping aiming for in the measurement of the long satellite base lines.

CHAIRMAN: We hope to get 1 p.p.m. after application of the geoidal corrections.

ANGUS-LEPPAN: Mather has been studying the orientation of the geoid with respect to the Australian National Spheroid (ANS). The separation naturally affects the scale of geodetic surveys. Can this accuracy be achieved in the light of our present knowledge of the geoid?

MATHER: An accuracy of 1 p.p.m. corresponds to a geoid-spheroid separation of 6 m. The geoid slope also affects accuracy. In Australia the separation might be up to 28-30 m.

DISCUSSION - GEODETIC MEASUREMENTS

LAMBERT: The deviation of the vertical does not affect the issue as the satellite is photographed against a stellar background. Base lines only provide the scale.* Mrs. Fisher has computed the geoid separation and the slope is only 10 secs.

FRYER: The close agreement between Mrs. Fisher's geoid and the ANS is no coincidence as the orientation of the ANS was chosen as a best mean fit of the spheroid to the geoid in the Australian region.

GALE: With our present lack of knowledge of the geoid I do not consider that an accuracy of 1/1 000 000 is possible.

JONES: There is a significant difference between the two sets of measurements of the Somerton Base as shown in Table I, Part A and Fig. 16.1. Was there any marked change in the meteorological conditions to account for this?

BOBROFF: The two results are significantly different and have been investigated. However I have been unable to find any factor causing the difference.

ROBINSON: Is there any significance in the fact that the (MRA 4 minus MRA 1) measurement differences are all positive while the (MRA 4 minus adjusted length) differences are all negative.

BOBROFF: The results are not really equivalent. The first difference is a direct comparison between measured distances, whereas the second is a comparison between measured distances and adjusted distances obtained not only from measured distances but also from triangulation.

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* The position of the base line is vital to scale as a correction for height above reference surface or 'sea-level' correction must be applied. Satellite observations are made independent of the direction of the vertical, but with respect to the earth's rotation axis. The position of the Australian National Spheroid with respect to the earth's axis and centre of gravity were unknown before Mather's studies using gravity. Ed.