

GMAT3100
DATUMS OF NETWORKS AND FREE NETWORKS

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It is normal in least squares adjustments to hold a certain number of parameters 'fixed' so that a solution becomes possible. One way around the use of "fixed points" is to use special solution techniques, such as the "free network" adjustment where none of the points are held fixed. Free network adjustments have been around for about 40 years, but they are rarely documented in detail. Often text books and journal papers that refer to freenets give the theory and equations but have limited space to discuss the details that students require to develop an understanding. Once the theory has been established that is fine, but there are still important matters to consider when implementing the theory in practice. In keeping with the style of Harvey (2009) here we use real student measurements in our examples and consider the practical implementation and interpretation of results. I include my opinions on the use and interpretation of these topics in places where decisions can be subjective.

A free network datum is also called "minimum trace datum" or "inner constraint datum". It allows a solution without having to fix any coordinate parameters to meet the rank defect of the network. A free network will yield a Q_x matrix with the smallest standard deviations and error ellipses. Smaller error ellipses are one of its attractions but that doesn't mean the results are more accurate – it is all a matter of correct interpretation.

Free Networks are sometimes used in deformation calculations to define the datum, but before discussing deformation these notes investigate free network solutions for control surveys in general.

We commence with some revision of least squares and datum issues, and introduce our symbols (following those used in Harvey, 2009). We use very small networks from student observations at Chifley Dam near Bathurst NSW as examples. In my research, but not in these notes, I use three different styles of freenet solution and compare their results as a check that I have correctly implemented the methods. I also compared using angles with using direction observations; if done correctly they both give the same answers. In these notes we use the Caspary (2000) method. A useful tutorial learning experience is to do the calculations of some of these examples and show one more decimal place in the answers and intermediate steps.

No apology is made for the length of this document; it is long because I want to show the details.

1. Revision of least squares and datums for control survey networks

For Parametric LS method

$$\Delta x = N^{-1}t = (A^T P A)^{-1} A^T P b = N^{-1} A^T P (\ell - f(x_a)) \quad \text{and} \quad Q_x = N^{-1}$$

The variance covariance matrix of the adjusted parameters, Q_x , depends on the datum, so error ellipses which are calculated from Q_x vary with the choice of origin and orientation of the network.

How many constraints (fixed parameters)? The number of zero eigenvalues of N is the rank defect and is the number of parameters that have to be held fixed.

Not enough constraints → crash (at inversion of N or solution of normal equations)

Minimum constraints → just enough to get a solution, but no distortion of internal structure of the network

Over constraints → possible distortions if there are errors in fixed coordinates

When a network of observed lines is adjusted with *minimum constraints*, the point error ellipses tend to get larger the further the point is from the fixed point. Adjusting the network with a different fixed point will change the point error ellipses. The relative error ellipse of a line between any two points in the network will not depend on which points define the datum in a minimally constrained solution. An example...

If you are going to *over constrain* a network then holding fixed points on opposite extremities of the net is better than holding points near each other at only one end of the network. That is because any scale errors can be monitored and controlled. As a by-product, the error ellipses will also be smaller if you interpolate inside fixed points instead of extrapolating from fixed points at one end of a network.

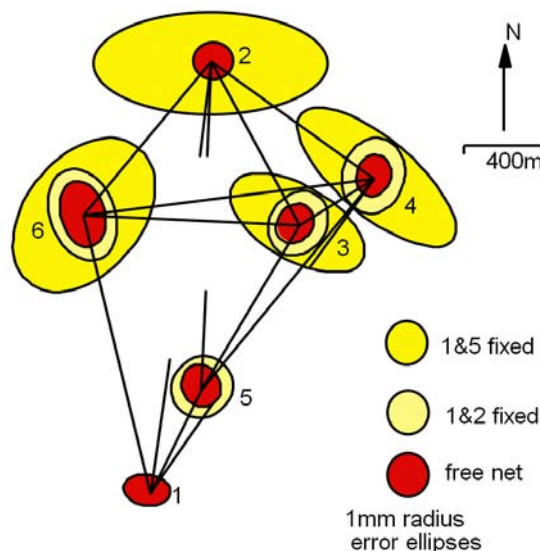
2. Overview of Free Networks and Graphical Interpretation

In free networks, the trace of Q_x ($\text{tr}(Q_x)$ = the sum of the diagonal terms of the matrix Q_x) is a minimum, and is sometimes called the inner precision. Similarly, the free net solutions X_f are sometimes referred to as inner coordinates. A free network solution can be derived from a minimum constraint solution or calculated directly by changing the Normal (N) matrix during a solution. The inner coordinates X_f are dependent on the input coordinates because the input / starting coordinates are used to calculate a free net.

In summary, a free network solution has the following properties:

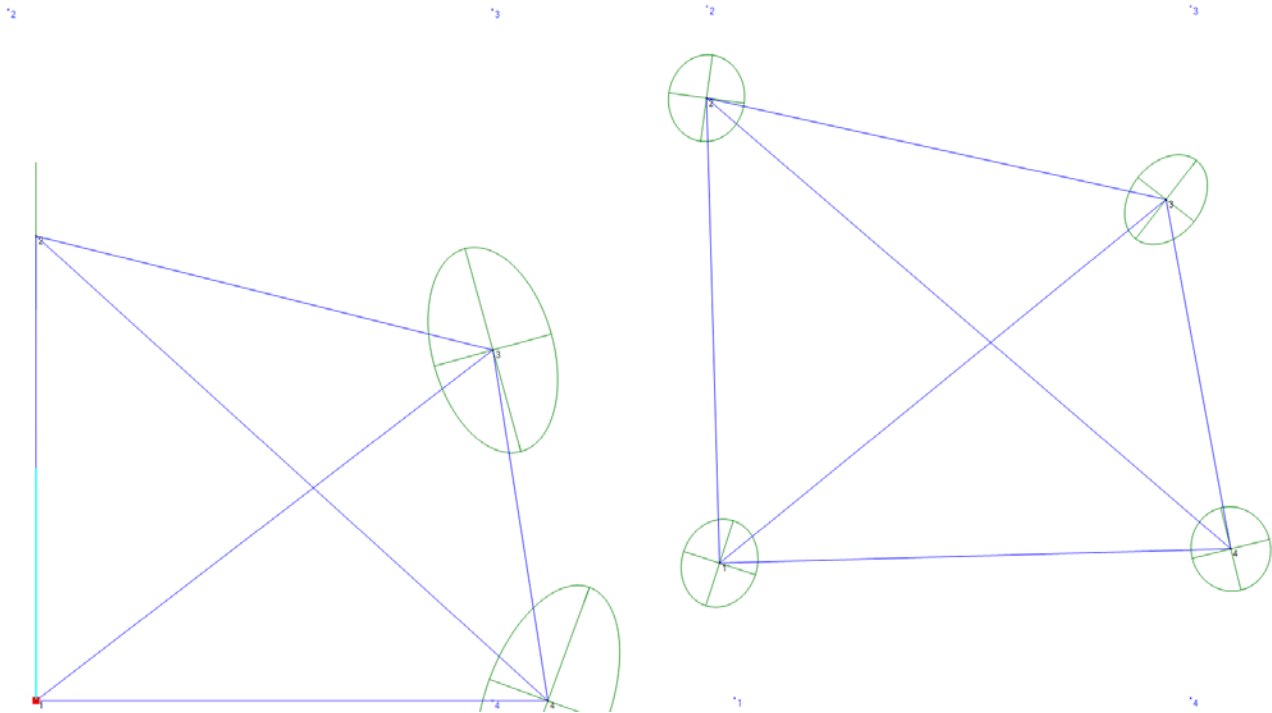
- No parameters or observations are held fixed
- Sum of weighted residuals squares ($v^T P v$) is a minimum ("least squares principle")
- Trace of the matrix Q_{X_f} is minimum.
- Norm $\Delta X_f = \sqrt{(\Delta X_f^T \Delta X_f)} = \sqrt{\Sigma(\Delta X_f^2)}$ is a minimum. Where ΔX_f are the corrections to the input coordinates.
- Freenet calculations can be carried out in several different ways, one way:
 - Do a conventional minimally constrained adjustment
 - Then transform the adjusted coordinates and their VCV
- The location and orientation (and sometimes scale) of the adjusted free network is provided by the starting coordinates. So changing the starting coordinates can change your results!
- The coordinates of the centre of the network do not change. The centre is not necessarily a surveyed point. The centre is calculated to be at the average of the coordinates. The centre (mean) of the starting coordinates is the same as the centre (mean) of the adjusted freenet coordinates: $(\Sigma E_s/n, \Sigma N_s/n, \Sigma H_s/n) = (\Sigma E_f/n, \Sigma N_f/n, \Sigma H_f/n)$
- Free net point error ellipses behave like relative error ellipses and don't contain large datum biases
- Applications of free nets:
 - Can be used in (high precision) engineering surveys with local datums
 - Can be used in some deformation analysis
 - Not used for control surveys onto state MGA coordinate systems (or similar)

Harvey (2009, Monograph 13 sec 9.3) uses the figure below to show that point ellipses from a freenet are smaller than from constrained solutions. The freenet equations were derived to do that. Notice also that when fixed points in a network are all at one edge, corner or extremity of a network (e.g. points 1 and 5 in the figure below) then the point error ellipses grow as you move across the net. If you have a choice to be able to hold any point fixed in a network (e.g. it is on arbitrary local coordinate datum) then choosing a point near the centre of the net will give smaller point ellipses – for the same data. That is not necessarily a better solution, but it looks better ☺.



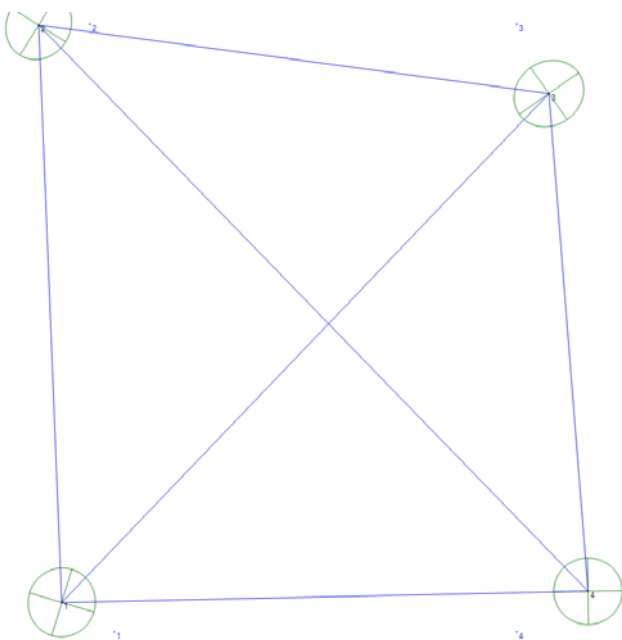
So, freenet adjustments affect the point error ellipses. What do freenet adjustments do to coordinates? Harvey (2009, Monograph 13 sec 1.6) uses a simple 'braced quad' of distance measurements to graphically illustrate the effect of least squares adjustments. This example grossly exaggerates the effects to show what happens to coordinates and ellipses. Here we show the results of minimally constrained adjustments and of free net adjustments of the same data. The starting coordinates are shown separate to the adjusted coordinates. The adjusted coordinates are linked by the observation lines and the error ellipses are centred on the adjusted coordinates, in the figures below.

A minimally constrained solution with one fixed point (1) and an azimuth (1 to 2) is shown at left below. As you can see the above network is 'locked' onto the coordinate of point 1 and rotated to fit the given azimuth. Later we discuss 3 and 4 parameter free net solutions for 2D data sets in detail, but let's look at some results first. A 3 parameter free net solution uses the starting coordinates of points to determine the location (shift, translation in E and N) and orientation (rotation) to fit the adjusted network to the 'centre' of the starting position, is shown at right below.



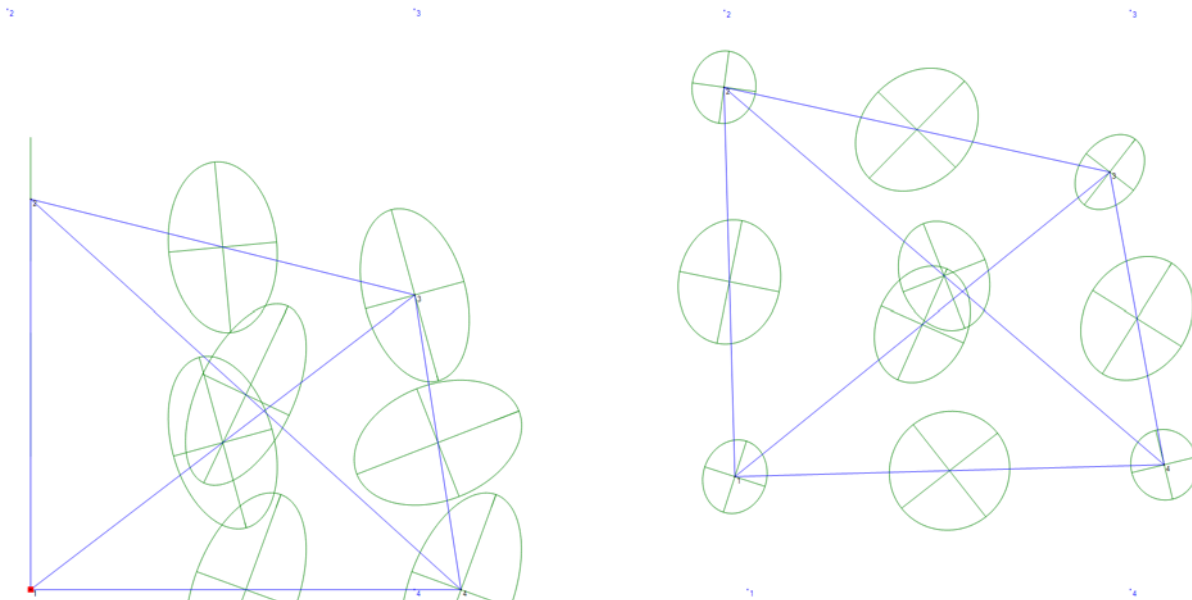
The adjusted distances and the residuals are the same in both methods shown above. If the starting coordinates come from a prior set of good quality observations that have been adjusted with constrained least squares, then the effect of a free net solution on the coordinates is much smaller than in the exaggerated example above.

A 4 parameter free net solution includes a scale parameter to stretch the adjusted network closer to the starting points, as shown in the figure below.



In both the 3 parameter and 4 parameter 2D free net solutions of this braced quad, the mean E and mean N of the starting coordinates is the same as the mean E and mean N of the adjusted free net coordinates. The centre of the net hasn't moved when we do a free net solution, but the centre of the network does move when we do a minimally constrained solution with point 1 fixed.

Let's briefly look at the relative error ellipses of the lines. On the left below, the minimally constrained solution with one fixed point (1) and an azimuth (1 to 2), with relative error ellipses is shown. On the right below, a 3 parameter free net solution with relative error ellipses is shown.



In summary, a free net gives small overall movement to the points. That is, it locates the network based on the starting coordinates of all (or sometimes a subset) of the points. Think of it as shifting and rotating the net, and sometimes stretching it. It gives smaller point error ellipses, but the same residuals and VF and similar relative error ellipses.

A free network is not necessarily a better solution just because the error ellipses are smaller. It depends on the application and on the circumstances of the survey project.

3. Example Free Network with height differences only

Our data comes from student survey camps in the early 1990s at Chifley Dam near Bathurst NSW. The 2D part is used extensively in Harvey (2009).

Point	Height (m)
7	828.020
1	746.0
5	704.0

Point 7 is a traditional trig station with AHD height known from previous surveyors. Points 1 and 5 are marks placed by students, with approximate height from pre-processing calculations.



Height differences were measured by total station using EDM and ZA, and taped hi, ht. Mean of forward and back observations:

From	To	Mean Δ Ht (m)	Estimated standard deviation based on length of line (mm)
1	5	-41.556	5
1	7	82.036	16
5	7	123.566	21

Loop misclose = 0.026 m

3.1 Minimum constraint solution

A conventional minimum constraint adjustment with H7 fixed at 828.020 has 2 coordinate parameters and 3 observations. The results from FIXIT are:

Adjusted Heights	Standard Deviations	Adjusted height - Input height
1 745.9932 m	12.9 mm	-6.8 mm
5 704.4381 m	13.1 mm	438.1 mm
7 828.0200 m	0.0 mm	0.0 mm
	$\Sigma S_H \approx 26 \text{ mm}$	$\Sigma \Delta h \approx 444.9 \text{ mm}$

Our Standard Deviations are NOT multiplied by vVF

RESIDUALS AFTER ADJUSTMENT							
OBS TYPE	FROM	TO	MEASUREMENT	STD DEV	v	v/s	Redun
Height Dif	1	5	-41.556	6.0mm	0.9mm	0.2	0.03
Height Dif	1	7	82.036	16.0mm	-9.2mm	-0.6	0.4
Height Dif	5	7	123.566	21.0mm	15.9mm	0.8	0.6

Variance Factor is 0.94 [n-u = 1, Ave redun = 0.33]

3.2 Freenet solution

In our usual least squares solutions we solved: $\Delta x = (A^T P A)^{-1} A^T P b$ and $Qx = N^{-1}$

where:

- A = partial derivatives matrix
- P = Q^{-1} , where Q is the VCV matrix of observations
- Δx = solution vector
- b = observed – calculated vector based on the observations

To obtain a solution for a free network with parametric least squares Caspary's method calculates the N matrix as usual, but with all coordinates treated as parameters. The main equations, with our symbols, are: (Caspary, 2000, pp18, 3-23): [Read the reference to see the derivation.]

$$Qx_F = (N + DD^T)^{-1} N(N + DD^T)^{-1} \Delta x = Qx_F (A^T P b)$$

where

- N = $A^T P A$
- D = Datum transformation matrix, see below

The D matrix is the part that deals with the datum issue. For a 1D height network we have to constrain or provide a datum for the heights. How to do this, and what it means, will become clearer later when we discuss 2D and 3D networks. For now, just accept that D is a column vector of 1s.

Using our example data; set up the matrices and vectors for a levelling adjustment as though we were going to solve for all 3 heights.

Observations, in order: $\Delta h_{15}, \Delta h_{17}, \Delta h_{57}$ Parameters, in order: h1, h5, h7

A	b in mm	P (units of s = mm)
-1 1 0	444.0	0.040 0 0
-1 0 1	16.0	0 0.004 0
0 -1 1	-454.0	0 0 0.002

Matrix multiplication gives:

$$N = A^T P A$$

0.044	-0.040	-0.004
-0.040	0.042	-0.002
-0.004	-0.002	0.006

$$A^T P b$$

-17.82
18.79
-0.97

The determinant of N is extremely close to zero. So we probably can't invert N. The eigenvalues of N are ...?

$N = A^T P A$	D	DD^T	$N + DD^T$	$(N + DD^T)^{-1}$
0.044 -0.040 -0.004 -0.040 0.042 -0.002 -0.004 -0.002 0.006	1 1 1	1 1 1 1 1 1 1 1 1	1.044 0.960 0.996 0.960 1.042 0.998 0.996 0.998 1.006	23.12 12.12 -34.91 12.12 25.26 -37.05 -34.91 -37.05 72.29

The determinant of the $N+DD^T$ matrix above is 0.002.

Q_{x_f} in mm^2	Δx in mm		Adjusted Heights, m	Standard devn's, mm
23.01 12.01 -35.02	-150.6	1	745.849	4.8
12.01 25.15 -37.16	294.3	5	704.294	5.0
-35.02 -37.16 72.18	-143.8	7	827.876	8.5

The standard deviations of the coordinates are the square roots of the diagonals of the inverse matrix, as usual in LS, e.g. $\sqrt{23.01} = 4.8$

The sum of the absolute values of the Δx terms is 588.7 mm. The sum of the standard deviations is 18.3mm which is smaller than the equivalent value (26mm) for the minimally constrained solution above.

Another iteration can be calculated using the same process as above, but this is a linear problem so Δx all equal zero, and the adjusted heights and their Q_{x_f} (and standard deviations) are the same.

Look at the 'inner' precision. By this free net solution the join between the adjusted heights of 1 and 5 is -41.5551m and its standard deviation by propagation of variances $[\sqrt{23.01+25.146-2*12.011}] = \pm 4.91$ mm. The FIXIT minimally constrained solution above gives exactly the same values! So neither solution is 'distorted', but the standard deviations of the point's heights by free net solution are smaller.

The details for all the lines:

Calculated Lines between adjusted coordinates

From	To	Height	Difference	+/-
1	5	-41.5551 m		4.91 mm
1	7	82.0268 m		12.85 mm
7	5	-123.5819 m		13.10 mm

So free net is about the error bars (or ellipses) of the points, not the relative error bars (or relative error ellipses) of lines between any two points in a network. Free net does not affect the internal characteristics of the network (unless you scale it – more details about that later). The other consequence of the free net solution is that the height of the trig station at 7 is now different to the 'published' value.

Consider the effect of the starting / input coordinates. If the starting coordinates are derived from an independent survey such as published state survey mark coordinates we might want to do a free net using those values for H_s . But if the marks are new with no prior known values then one option is to compute a minimally constrained adjustment and use the final coordinates as our starting values then the fixed point won't change its height? For the above data set if we use the adjusted heights from a minimally constrained solution with H7 fixed, as the starting values then:

H_s m		H_f m
745.993	H1	745.993
704.438	H5	704.438
828.020	H7	828.020

The above approach might be useful in some applications.

Another case to consider for the starting coordinates for free net adjustments is when we have multiple epochs of the network in a deformation or monitoring survey. If there are multiple epochs then we might wish to use the same starting coordinates (H_s) for each epoch.

3.3 Residuals, degrees of freedom (n-u), and VF for each of the methods

For the minimally constrained adjustment there are 3 observations (n) and 2 unknowns, so $n-u = 1$ the residuals are 0.9, -9.2, and 15.9mm. The Variance Factor is 0.94.

For the free net solution the residuals (v) are the same as the minimally constrained method. [Calculated from e.g. $v_1 = (\text{adjusted 'free' height } 5 - \text{adjusted 'free' height } 1) - \text{obs } \Delta H_{15} = 0.9$]

There are 3 observations (n) but how do we count the number of unknowns in a free net solution? I use $u = \text{number of coordinate parameters} - \text{number of columns (datum constraints) in } D$. In this example $u = 3-1 = 2$. That is, the u is the same for both free and minimally constrained adjustments. So VF is the same for both methods.

4. Revision of 2D and 3D similarity transformations

Before we learn about Free net adjustments for 2D and 3D networks it is helpful to revise transformation parameters. Start with the equations for similarity transformations. A similarity transformation shifts and rotates a network as a whole; it does not change the internal geometry of a network. That is, the angles between lines do not change and the relative lengths of lines do not change but there can be an overall scale factor applied. A 2D similarity transformation from e, n coordinates to E, N coordinates has equations:

$$\begin{aligned} E &= se \cos \phi + sn \sin \phi + Te \\ N &= -se \sin \phi + sn \cos \phi + Tn \quad \text{or} \quad \begin{pmatrix} E \\ N \end{pmatrix} = s \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} e \\ n \end{pmatrix} + \begin{pmatrix} Te \\ Tn \end{pmatrix} \end{aligned}$$

Some textbooks use x y coordinates. We use E and N and the rotation ϕ is clockwise from the N axis like a bearing.

In a 2D network a similarity transformation usually has 4 parameters, being 2 shifts, a rotation and a scale factor. However if reliable distance observations are included in the network then 3 transformation parameters (2 shifts and a rotation) may be sufficient. In a 1D network like levelling there is a single shift/translation parameter.

A free network can be thought of as a similarity (or conformal or S or Helmert) transformation of a minimally constrained solution of a network. So the questions arise: where do I transform from and to, and how many parameters? When doing a free network we can transform the adjusted coordinates of a minimally constrained solution and their Qx matrix (and thus error ellipses and standard deviations of coordinates) or we can build the transformation into the solution process. Also, we can choose to determine the transformation parameters from all points or from just some of the points such as the most well known or the most stable points in a deformation study.

Generally the transformations from starting coordinates to final free net coordinates involve small changes in position. Since the rotation parameters in free net calculations are small the approximations $\sin \phi = \phi$ (in radians) and $\cos \phi = 1$ can be made. Here we assume all e, n coordinates of all points have equal precision (or equal weight), that is all points contribute equally to the datum. Later we discuss how to use a subset of points, but even then all points within the subset are treated equally.

If ϕ is small and if we replace s with $s = 1+u$, where u is a small term and $u * \phi$ is close enough to zero to ignore, then the 2D transformation formula above can be rearranged to:

$$\begin{pmatrix} E \\ N \end{pmatrix} = \begin{pmatrix} e \\ n \end{pmatrix} + \begin{pmatrix} e & n & 1 & 0 \\ n & -e & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ \phi \\ Te \\ Tn \end{pmatrix}$$

The transformation formula can be further simplified if ϕ is small and if s (scale) = 1 and $u = 0$ (i.e. we choose to not have a scale parameter):

$$\begin{pmatrix} E \\ N \end{pmatrix} = \begin{pmatrix} e \\ n \end{pmatrix} + \begin{pmatrix} n & 1 & 0 \\ -e & 0 & 1 \end{pmatrix} \begin{pmatrix} \phi \\ Te \\ Tn \end{pmatrix}$$

These matrices above lead to the D matrix for 2D datum in the Caspary and other free net methods. When working with a 2D traversing network with angle, or direction, and distance observations the network has a rank defect of 3 and so I choose a D matrix with 3 parameters, namely a rotation, a shift / translation in E, and a shift in N. This follows the concept that the scale parameter is defined or fixed by the distance measurements, and that free networks are defined by minimal constraints. If there are no distance observations in a 2D network, use a 4 parameter transformation. There is a subjective decision to be made about whether to include a scale parameter or not. Some people may prefer to always include a scale parameter, some do not. My opinion is that if there are reliable distance observations in the network then I do not usually include a scale parameter. Some people prefer to use a 4 parameter free net model for all 2D networks, whether they include distance observations or not.

An example of the consequences of choosing 4 parameters (including scale) follows. If you adjust a network with minimum constraints calculate the joins of some of the lines, i.e. distances and bearings and their standard deviations from adjusted coordinates. Then do a free net transformation of the adjusted coordinates onto the input coordinates with 4 parameter model, this will usually scale (stretch/shrink) the adjusted net to the input coordinates. Then calculate the joins of the lines from the free net coordinates. The distances will usually change because of the scale; the standard deviations of the distances will be OK. So the free net is being stretched (shrunk) to fit the input coordinates. The decision then depends on how much you trust the scale of the input coordinates or the scale of your distances as to whether you want this stretching or not.

In a 2D network with coordinates in the order E then N, for p points and including distance observations, with datum parameters in the order ϕ, Te, Tn :

$$x = \begin{pmatrix} e_1 \\ n_1 \\ e_2 \\ n_2 \\ \vdots \\ e_p \\ n_p \end{pmatrix} \quad D = \begin{pmatrix} n_1 & 1 & 0 \\ -e_1 & 0 & 1 \\ n_2 & 1 & 0 \\ -e_2 & 0 & 1 \\ \vdots & \vdots & \vdots \\ n_p & 1 & 0 \\ -e_p & 0 & 1 \end{pmatrix}$$

Note that the order of the parameters determines the order of the columns in D. But it doesn't matter which order the columns in D are placed in. In a 2D network without distance observations 4 parameters are needed, if the parameters are in the order u, ϕ, Te, Tn then:

$$D = \begin{pmatrix} e_1 & n_1 & 1 & 0 \\ n_1 & -e_1 & 0 & 1 \\ e_2 & n_2 & 1 & 0 \\ n_2 & -e_2 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ e_p & n_p & 1 & 0 \\ n_p & -e_p & 0 & 1 \end{pmatrix}$$

In 3D networks with e, n, h coordinates, there are 7 transformation parameters: a small scale change (u), small rotations (Φ_e, Φ_n, Φ_h), and small shifts (Te, Tn, Th). If they are used in that order then the following D matrix applies:

$$D = \begin{pmatrix} e_1 & 0 & -h_1 & n_1 & 1 & 0 & 0 \\ n_1 & h_1 & 0 & -e_1 & 0 & 1 & 0 \\ h_1 & -n_1 & e_1 & 0 & 0 & 0 & 1 \\ etc \end{pmatrix}$$

If reliable distances are observed in the network and you decide not to solve for scale then omit column 1 in D above. Similarly, if a theodolite or other instrument with reliable vertical sensor is used for directions then omit the columns for rotations Φ_e and Φ_n , in D, then:

$$D = \begin{pmatrix} n_1 & 1 & 0 & 0 \\ -e_1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ n_2 & etc \end{pmatrix}$$

For completeness let's revisit 1D networks (e.g. levelling) as described in an early section. The datum transformation matrix is a subset of the 3D system, for a single datum parameter being a shift in height:

$$D = (1, 1, 1, \dots, 1)^T$$

5. Example 2D Network

This section follows on the 1D network procedures above, but now with distances and directions we have non linear LS problems and have to consider iterations in our solution. We also have more terms in our D matrix, and that brings with it some additional questions.

The Chifley Dam data set used in Harvey (2009, p118) in FIXIT input format follows. The coordinates of trig stations 7 and 12 are known from government survey data and the coordinates of 1 and 5 are approximate values from prior calculations. The coordinates are on a projection system but have been truncated here. The H DIS values are the appropriately corrected grid/projection distances.

TITLE	PART OF CHIFLEY DAM INPUT FILE				
DSD	0.9	0.0	1.5	0.0	
COORDINATE	1	Rod	9279.		5154.
COORDINATE	5	B Bolt	8794.		4889.
COORDINATE	7 EN	Windy Corner	10064.072		6612.433
COORDINATE	12 EN	Ovens	22099.220		21416.713
H DIS	1	5	552.968		
H DIS	5	7	2139.950		
H DIS	1	7	1655.179		
DIRECTION	1	5	40 47 30.		
DIRECTION	1	7	187 43 19.		
DIRECTION	5	7	179 59 52.		
DIRECTION	5	1	204 57 35.		
DIRECTION	7	12	139 17 49.		
DIRECTION	7	1	308 28 22.		
DIRECTION	7	5	316 34 43.		

Consider a minimum constraint solution for this data. With a 1D heighting network it is simply a matter of choosing 1 height to hold fixed for a minimum constraint solution. But how do we get a minimum constraint solution for our 2D triangle? We want 3 coordinate parameters to be fixed. An E and N of one point is the usual method for the first 2 parameters. The third parameter could be constrained by:

- Holding the E or the N of one of the other points fixed, but unless one of the lines in the network is oriented EW or NS then that choice seems a bit arbitrary and artificial.
- Hold the azimuth of one line fixed as well as one point. An azimuth can be held fixed by including one extra observation for a line, being an azimuth observation (with no orientation parameter). That will affect the count of the number of observations. So n-u and thus VF will be affected. The A P and b matrices gain a row.

- c) Include angle or direction observations to another point that is held fixed and only observed to from one point for orientation purposes. In our Chifley dam triangle that could be the direction to point 12, and hold the coordinates of 12 and 7 fixed. If point 12 is included in our solution and becomes part of the freenet transformation, the freenet coordinates and Q_x will also be affected.

Here we will use option (a) above, because it can be more closely compared with our freenet solutions that follow. I choose to hold E7 N7 E1 fixed for our minimally constrained solution of this triangle, and not to use observations to point 12.

The standard least squares solution after iteration has led to suitable convergence, yields Q_x for the parameters in order N1 E5 N5 Ω_1 Ω_5 Ω_7 :

Q_x units are mm^2

0.900	0.457	-0.178	-0.191	-0.240	0.004
0.457	3.157	-2.925	-0.822	-1.042	-0.193
-0.178	-2.925	3.672	0.886	1.110	0.213
-0.191	-0.822	0.886	1.369	0.307	0.052
-0.240	-1.042	1.110	0.307	1.511	0.065
0.004	-0.193	0.213	0.052	0.065	1.139

Adjusted Coordinates after sufficient iterations

Point	Fixed	E	N
1	E	9279.0000	5155.2858
5		8793.5786	4890.4474
7	EN	10064.0720	6612.4330

The residuals in the same order as the observations are: -0.3, 0.4, -0.4 mm, -0.9, 0.9, -3.0, 3.0, 0.5, -0.5" and $VF = 3.24$

5.1 Caspary, 2D, 3 parameter Freenet

In this section we consider an adjustment of the 1 – 5 – 7 triangle since point 12 is purely an orientation target. To start with let's use directions and distances in the triangle and solve the Freenet by Cooper's method for 3 datum transformation parameters. This will show us how to deal with orientation parameters (or any other non coordinate parameter).

Data:

Point	E	N
1	9279.000	5154.000
5	8794.000	4889.000
7	10064.072	6612.433

	obs	std dev	calc dis/Brg°	Calc Ω° (= bearing-direction for first target)
Dis 15	552.968 m	0.9 mm	552.675	
Dis 57	2139.950 m	0.9 mm	2140.865	
Dis 17	1655.179 m	0.9 mm	1656.311	
Dir 15	40° 47' 30"	1.5"	241.348	200.55650
Dir 17	187° 43' 19"	1.5"	28.294	
Dir 57	179° 59' 52"	1.5"	36.388	216.39032
Dir 51	204° 57' 35"	1.5"	61.348	
Dir 71	308° 28' 22"	1.5"	208.294	259.82073
Dir 75	316° 34' 43"	1.5"	216.388	

From the data: Ang misclose around the loop 1 – 5 – 7 = 7.0"

Calculate the partial derivative (A), observed-calculated (b) and quality of observations ($Q \rightarrow P$) matrices assuming none of the points are held fixed (i.e. with columns for all coordinates). We use the observations in the order above, and parameters in the order e1, n1, e5, n5, e7, n7. As in Harvey (2009) the distance partials are unit less; and standard

deviations and b are in the same units i.e. mm and “. The partials for directions are multiplied by the radians to seconds and m to mm conversion factor: 206.2648...

To save space only the diagonal terms of P are shown, the other terms are 0.

A: E1	N1	E5	N5	E7	N7	Ω1	Ω5	Ω7
0.878	0.479	-0.878	-0.479	0	0	0	0	0
0	0	-0.593	-0.805	0.593	0.805	0	0	0
-0.474	-0.881	0	0	0.474	0.881	0	0	0
0.179	-0.328	-0.179	0.328	0	0	-1	0	0
-0.110	0.059	0	0	0.110	-0.059	-1	0	0
0	0	-0.078	0.057	0.078	-0.057	0	-1	0
0.179	-0.328	-0.179	0.328	0	0	0	-1	0
-0.110	0.059	0	0	0.110	-0.059	0	0	-1
0.0	0.0	-0.078	0.057	0.078	-0.057	0	0	-1

b
292.7
-915.3
-1131.6
0.0
-54.2
0.0
6.7
0.0
40.5

Pij
1.23
1.23
1.23
0.44
0.44
0.44
0.44
0.44
0.44
0.44

$$D = \begin{pmatrix} n_1 & 1 & 0 \\ -e_1 & 0 & 1 \\ n_5 & 1 & 0 \\ -e_5 & 0 & 1 \\ n_7 & 1 & 0 \\ -e_7 & 0 & 1 \end{pmatrix}$$

The D matrix with 3 datum parameters in the order rotation, shift E, shift N:

Each row in D above relates to the parameters in the order of the coordinates used to create the A matrix. Then add row of 0 terms for each orientation parameter. Though I haven't seen any reference books mention what to do with the orientation terms.

N = A ^T PA								
1.27	0.98	-0.98	-0.47	-0.29	-0.51	-0.03	-0.08	0.05
0.98	1.34	-0.47	-0.38	-0.51	-0.96	0.12	0.15	-0.03
-0.98	-0.47	1.42	1.05	-0.44	-0.59	0.08	0.11	0.03
-0.47	-0.38	1.05	1.18	-0.59	-0.80	-0.15	-0.17	-0.03
-0.29	-0.51	-0.44	-0.59	0.73	1.10	-0.05	-0.03	-0.08
-0.51	-0.96	-0.59	-0.80	1.10	1.76	0.03	0.03	0.05
-0.03	0.12	0.08	-0.15	-0.05	0.03	0.89	0	0
-0.08	0.15	0.11	-0.17	-0.03	0.03	0	0.89	0
0.05	-0.03	0.03	-0.03	-0.08	0.05	0	0	0.89

D		
5154	1	0
-9279	0	1
4889	1	0
-8794	0	1
6612.433	1	0
-10064.072	0	1
0	0	0
0	0	0
0	0	0

A ^T Pb		
982.5		
1401.0		
351.3		
738.4		
-1333.8		
-2139.4		
24.1		
-3.0		
-18.0		

As expected, N above can't be inverted by the common methods, its determinant is zero.

DD^T for 2D data is not as small or as simple as for 1D data. So we create and store D then do the matrix multiplications.
DD^T =

2.66E+07	-4.78E+07	2.52E+07	-4.53E+07	3.41E+07	-5.19E+07	0	0	0
-4.78E+07	8.61E+07	-4.54E+07	8.16E+07	-6.14E+07	9.34E+07	0	0	0
2.52E+07	-4.54E+07	2.39E+07	-4.30E+07	3.23E+07	-4.92E+07	0	0	0
-4.53E+07	8.16E+07	-4.30E+07	7.73E+07	-5.81E+07	8.85E+07	0	0	0
3.41E+07	-6.14E+07	3.23E+07	-5.81E+07	4.37E+07	-6.65E+07	0	0	0
-5.19E+07	9.34E+07	-4.92E+07	8.85E+07	-6.65E+07	1.01E+08	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

N+DD^T below can be inverted by the common methods; its determinant is 7.4E+7.

N+DD ^T								
2.66E+07	-4.78E+07	2.52E+07	-4.53E+07	3.41E+07	-5.19E+07	-0.03	-0.08	0.05
-4.78E+07	8.61E+07	-4.54E+07	8.16E+07	-6.14E+07	9.34E+07	0.12	0.15	-0.03

(N+DD ^T) ⁻¹								
4.46	-2.04	5.15	-4.51	-8.41	5.08	-1.38	-1.27	-1.72
-2.04	1.56	-1.61	1.21	3.43	-2.06	0.31	0.19	0.70

2.52E+07	-4.54E+07	2.39E+07	-4.30E+07	3.23E+07	-4.92E+07	0.08	0.11	0.03	5.15	-1.61	9.57	-8.74	-12.94	7.90	-2.83	-2.91	-2.62
-4.53E+07	8.16E+07	-4.30E+07	7.73E+07	-5.81E+07	8.85E+07	-0.15	-0.17	-0.03	-4.51	1.21	-8.74	8.60	11.97	-7.32	2.74	2.85	2.41
3.41E+07	-6.14E+07	3.23E+07	-5.81E+07	4.37E+07	-6.65E+07	-0.05	-0.03	-0.08	-8.41	3.43	-12.94	11.97	19.36	-11.49	3.77	3.73	3.89
-5.19E+07	9.34E+07	-4.92E+07	8.85E+07	-6.65E+07	1.01E+08	0.03	0.03	0.05	5.08	-2.06	7.90	-7.32	-11.49	7.19	-2.29	-2.28	-2.35
-0.03	0.12	0.08	-0.15	-0.05	0.03	0.89	0	0	-1.38	0.31	-2.83	2.74	3.77	-2.29	2.01	0.93	0.76
-0.08	0.15	0.11	-0.17	-0.03	0.03	0	0.89	0	-1.27	0.19	-2.91	2.85	3.73	-2.28	0.93	2.11	0.75
0.05	-0.03	0.03	-0.03	-0.08	0.05	0	0	0.89	-1.72	0.70	-2.62	2.41	3.89	-2.35	0.76	0.75	1.91

Now we use the Caspary (2000) equations.

$$Qx_F = (N + DD^T)^{-1}N(N + DD^T)^{-1} \quad \Delta x = Qx_F(A^T Pb)$$

Qx _F in mm ² and '' ²									Δx
0.86	-0.75	-0.57	0.68	-0.29	0.07	0.27	0.39	-0.06	336.6 mm
-0.75	1.02	0.49	-0.82	0.26	-0.20	-0.32	-0.44	0.06	325.5 mm
-0.57	0.49	0.43	-0.38	0.14	-0.11	-0.18	-0.25	0.04	45.9 mm
0.68	-0.82	-0.38	0.77	-0.30	0.05	0.28	0.38	-0.06	247.6 mm
-0.29	0.26	0.14	-0.30	0.15	0.04	-0.10	-0.14	0.02	-382.5 mm
0.07	-0.20	-0.11	0.05	0.04	0.15	0.05	0.06	-0.01	-573.1 mm
0.27	-0.32	-0.18	0.28	-0.10	0.05	1.23	0.15	-0.02	27.5 "
0.39	-0.44	-0.25	0.38	-0.14	0.06	0.15	1.33	-0.03	16.7 "
-0.06	0.06	0.04	-0.06	0.02	-0.01	-0.02	-0.03	1.13	-26.3 "

The adjusted coordinates and their standard deviations are:

	E (m)	SD in mm	N (m)	SD in mm	Adjusted Ω°
1	9279.337	0.93	5154.325	1.01	200.57176
5	8794.046	0.65	4889.248	0.88	216.39962
7	10063.689	0.38	6611.860	0.38	259.80612

Where E1 = 9279.000 + 336.6/1000 and its standard deviation = √0.86 (from the relevant diagonal of the above matrix), and similarly for the other terms.

Unlike the 1D heighting example, this problem is not linear and we need to iterate. Using the above adjusted coordinates as our starting values and constructing all the matrices again from there raises a question. Should we change the coordinate values in D for each iteration or use the first set of starting coordinates? In practice it may not make much difference as long as the starting coordinates are not poor estimates of the final position. In these notes we will use the same D matrix in each iteration. Other authors may have different opinions and may choose to use the adjusted values after each iteration.

We also need to consider what units to use for the coordinates in D, but more about that later. If you are concerned about the large numbers in DD^T due to the coordinates, then simply divide the coordinates in the relevant column of D by 1000 i.e. enter coordinates in km, or some other factor. The relative values are weights in D are important, the magnitudes of these terms may not be as important. Try it yourself and see if the Qx_F and Δx are the same when you change the units of the coordinate terms in D. Since I create a Normals matrix (N) with terms that are in units of mm² in these examples and in my program, perhaps the coordinates in D should be in mm then in DD^T they are mm² too. But when I do so (use e.g. 5154000 for the first term) the solution for the some data sets crashes.

When we iterate the solution we use the adjusted parameters, that is adjusted coordinates E N and the adjusted values of the orientation parameters for directions. When we change the coordinates at each iteration we need to change the orientation parameters too. Otherwise we will get incorrect residuals and v/s, and therefore the VF will be wrong.

Our second iteration yields Δx terms of less than 0.1 millimetres. If our starting coordinates had not been so close to the final values then we might have needed more iterations. The converged freenet coordinates (the standard deviations of the coordinates are not much different to those above) are:

	Adj E	Adj N
1	9279.3366	5154.3255
5	8794.0459	4889.2476
7	10063.6895	6611.8599

Trace of coordinates part of $Q_{X_f} (=L^{-1}) = 3.37 \text{ mm}^2$

Norm of coordinate shifts = $\sqrt{(\Delta X_f^T \Delta X_f)} = \sqrt{(\Sigma(\Delta X_f^2))} = 0.87 \text{ m}$ where $\Delta X_f = X_f - X_s$

From the adjusted coordinates we can calculate the residuals: -0.3, 0.4, -0.4, mm -0.9, 0.9, -3.0, 3.0, 0.5, -0.5". There are 9 observations, 6 coordinate parameters, 3 orientation parameters and 3 datum parameters so the degrees of freedom (n-u) for use in VF calculation is 9-9+3 = 3, VF = 3.24.

Doing a freenet computation does not 'move' the overall position of the network, it just 'shuffles' the individual points coordinates to best fit with the observations. This can be proven by calculating the centre (mean) of the starting coordinates and seeing that it is the same as the centre (mean) of the adjusted freenet coordinates. Also, if you solve for 3 transformation parameters (1 rotation and 2 shifts) between the starting and the adjusted freenet coordinates, then the results are that there is no rotation and no shifts from the overall position of the network as defined by the starting coordinates to the overall position of the network as defined by the adjusted freenet coordinates (if we assume all coordinates have equal precision). This is a check calculation, but it also serves as prove of the effect of freenets and how they constrain the datum. If we had solved for a 4 parameter Datum freenet as in section 5.2 below then similarly the centre of the coordinates does not move and the transformation between starting and final coordinates equals zero rotation, scale factor equal 1, and zero shifts.

5.2 Caspary 2D, freenet with 4 Datum transformation parameters

Without going into all the details, here we show the effect of adding a scale transformation to the datum. If the parameters are in the order u, \emptyset , Te, Tn then:

$$D = \begin{pmatrix} e_1 & n_1 & 1 & 0 \\ n_1 & -e_1 & 0 & 1 \\ e_2 & n_2 & 1 & 0 \\ n_2 & -e_2 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ e_p & n_p & 1 & 0 \\ n_p & -e_p & 0 & 1 \\ 0 & 0 & 0 & \text{etc} \end{pmatrix} = \begin{pmatrix} 9279 & 5154 & 1 & 0 \\ 5154 & -9279 & 0 & 1 \\ 8794 & 4889 & 1 & 0 \\ 4889 & -8794 & 0 & 1 \\ 10064.072 & 6612.433 & 1 & 0 \\ 6612.433 & -10064.1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

a row of zeros for each orientation parameter

Iteration: 1 had a max shift in a coordinate = 548.4mm

Iteration: 2 had a max shift in a coordinate = 5.6mm

Iteration: 3 had a max shift in a coordinate = 0.1mm

Iteration: 4 had a max shift in a coordinate = 0.0mm

Adjusted Coordinates after iteration 4

Point	E m	N m
1	9279.3688	5154.3445
5	8794.0781	4889.2667
7	10063.7217	6611.8790

Residuals, with observations in the same order as before: -0.3mm, 0.4mm, -0.4mm, -0.9", 0.9", -3.0", 3.0", 0.5", 0.5"

The Variance Factor is 2.43 [n-u = 4, Ave redun = 0.44]

The coordinate and standard deviation values are considerably different to those obtained from the 3 parameter datum transformation, shown previously in section 5.1. So your decision about whether to use 3 or 4 parameters does make a difference to the results. Generally, more parameters will give smaller residuals (and VF) and smaller standard deviation for parameters, but that doesn't always mean it is a better solution. Which do you trust more to control the scale of your network, the starting coordinates or your measured distances? The answer to that may influence whether you apply a 3 or 4 parameter D matrix in the freenet solution.

Interestingly it is possible to do a freenet adjust of this triangle without using any of the distance measurements. If we tried to solve this triangle without distance observations in a normal Least Squares solution we would need to hold two points fixed (e.g. E1 N1 N7 N7) for a minimally constrained solution. If we use the 3 angle observations, and a 4 parameter D matrix that includes scale, then the freenet solution for the first iteration using the above data is:

E	sd mm	N	sd mm
9279.319	14.80	5154.158	10.35
8793.738	12.43	4888.914	6.38
10064.014	2.38	6612.361	4.01

Try it yourself.

5.3 Relative error ellipses

For the minimally constrained solution shown in section 5 above the point error ellipses in mm are [these are NOT multiplied by \sqrt{VF}]:

Point	S_E	S_N	Semi-major	Semi-minor	Bearing °
1	0	0.9	0.9	0.0	0
5	1.8	1.9	2.5	0.7	138
7	0	0	0	0	0

The Caspary 3 parameter Freenet solution yields:

Point	S_E	S_N	Semi-major	Semi-minor	Bearing °
1	0.9	1.0	1.3	0.4	138
5	0.7	0.9	1.0	0.4	147
7	0.4	0.4	0.4	0.3	46

The calculated join lines between the adjusted coordinates and their Qx from the minimally constrained solution are:

From	To	Distance	+/-	Bearing	+/-	Relative error ellipse in mm		
		m	mm	D M S	"	Semi-maj	Semi-min	Brg °
1	5	552.9677	0.85	241 23 01.7	1.0	2.7	0.7	142
5	7	2139.9504	0.83	36 25 12.6	0.2	2.5	0.7	138
1	7	1655.1786	0.84	28 18 52.6	0.1	0.9	0.0	0

The calculated join lines between the Caspary 3 parameter Freenet coordinates and their Qx (section 4.3) are:

From	To	Distance	+/-	Bearing	+/-	Relative error ellipse in mm		
		m	mm	D M S	"	Semi-maj	Semi-min	Brg °
1	5	552.9677	0.85	241 21 19.9	0.8	2.3	0.8	141
5	7	2139.9504	0.83	36 23 30.9	0.1	0.9	0.5	8
1	7	1655.1786	0.84	28 17 10.8	0.2	1.6	0.7	135

How do these compare? As we can see the distances are the same between the freenet (with no scale parameter) and minimum constraint solutions. The bearings of the lines differ by $-101.7''$, which is the amount of rotation of the network. The standard deviations of the length of the lines are the same. The standard deviations of the bearings and the relative error ellipses are similar but are affected by the rotation of the network.

The Caspary 4 parameter Freenet solution yields the same point error ellipses; and the same relative error ellipses and joins as the Caspary 3 parameter Freenet solution.

Our example network has only 3 lines and 3 points. In a larger network it is possible to calculate the relative error ellipses between any two points in the network; they do not have to be measured lines.

5.7 Partial Datum

It is possible to use a subset of points as datum points, for a free network. To achieve this, replace the relevant rows in the D matrix with zeros for those points that are not required to be part of the datum. Make sure that there are enough non zero rows in D to constrain the datum (ie at least the same number you would use in a minimum constraint solution). Our triangle network has too few points to demonstrate the application of a partial datum.

6. 3D Networks

To keep the notes short I give only a brief overview of 3D freenets. The procedure follows that given above.

6.1 If you combine the Chifley Dam triangle data for points 1 5 7 with height diffs, directions and grid distances, a freenet solution follows similar to the 2D method. I would suggest using a D matrix that has 4 columns for 1 rotation in azimuth and 3 shifts, but no scale parameter.

$$D = \begin{pmatrix} n_1 & 1 & 0 & 0 \\ -e_1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ n_2 & \text{etc} & & \end{pmatrix}$$

If parameters are in the order E1 N1 H1 E5 N5 H5 E7 N7 H7 Ω 1 Ω 5 Ω 7 there will be 9 rows in D in above pattern followed by three rows of zeros.

6.2 A different 3D case is to take the GPS vector example in Harvey (2009, Monograph 13, sec 4.12.5). There are 12 observations, P is not a diagonal matrix, and there are 12 coordinate parameters. A freenet solution with datum defined by D that has columns for 3 shifts only:

Starting values:

	X	Y	Z
TS	-4595099.898	2701464.208	-3492183.223
48	-4594852.600	2701461.000	-3492463.800
49	-4594909.900	2701376.800	-3492440.200
50	-4594980.200	2701340.000	-3492405.800

	obs m	b mm	Q mm ²																					
$\Delta X_{TS\ 48}$	258.939	11641	9	-0.399	2.13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
$\Delta Y_{TS\ 48}$	-2.801	407	-0.399	1	-0.54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Delta Z_{TS\ 48}$	-284.482	-3905	2.13	-0.54	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Delta X_{49\ 48}$	74.913	17613	0	0	0	4	-0.806	1.772	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Delta Y_{49\ 48}$	77.165	-7035	0	0	0	-0.806	1	-0.214	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Delta Z_{49\ 48}$	-29.551	-5951	0	0	0	1.772	-0.214	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Delta X_{49\ 50}$	-52.69	17610	0	0	0	0	0	0	1	-0.522	0.674	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Delta Y_{49\ 50}$	-43.871	-7071	0	0	0	0	0	0	-0.522	1	-0.333	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Delta Z_{49\ 50}$	28.531	-5869	0	0	0	0	0	0	0.674	-0.333	1	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Delta X_{TS\ 50}$	131.335	11637	0	0	0	0	0	0	0	0	0	9	-1.911	4.488	0	0	0	0	0	0	0	0	0	0
$\Delta Y_{TS\ 50}$	-123.835	373	0	0	0	0	0	0	0	0	0	0	0	0	-1.911	1	-1.576	0	0	0	0	0	0	0
$\Delta Z_{TS\ 50}$	-226.408	-3831	0	0	0	0	0	0	0	0	0	0	0	0	4.488	-1.576	4	0	0	0	0	0	0	0

The miscloses in X, Y, Z are 0.001, -0.002, 0.008 m respectively. A freenet solution with datum defined by D that has columns for 3 shifts would use D as follows:

	A partials, parameters in order Xts,Yts,Zts,X48, etc										D					
$\Delta X_{TS\ 48}$	-1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0
$\Delta Y_{TS\ 48}$	0	-1	0	0	1	0	0	0	0	0	0	0	0	0	1	0
$\Delta Z_{TS\ 48}$	0	0	-1	0	0	1	0	0	0	0	0	0	0	0	0	1
$\Delta X_{49\ 48}$	0	0	0	1	0	0	-1	0	0	0	0	0	0	1	0	0
$\Delta Y_{49\ 48}$	0	0	0	0	1	0	0	-1	0	0	0	0	0	0	1	0
$\Delta Z_{49\ 48}$	0	0	0	0	0	1	0	0	-1	0	0	0	0	0	0	1
$\Delta X_{49\ 50}$	0	0	0	0	0	0	-1	0	0	0	1	0	0	1	0	0
$\Delta Y_{49\ 50}$	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	1	0
$\Delta Z_{49\ 50}$	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	1
$\Delta X_{TS\ 50}$	-1	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0
$\Delta Y_{TS\ 50}$	0	-1	0	0	0	0	0	0	0	0	0	1	0	0	1	0
$\Delta Z_{TS\ 50}$	0	0	-1	0	0	0	0	0	0	0	0	0	1	0	0	1

Free Net Adjusted Values

	X m	± mm	Y m	± mm	Z m	± mm
TS	-4595104.226	1.5	2701462.153	0.5	-3492181.801	0.7
48	-4594845.285	1.2	2701459.352	0.5	-3492466.284	0.6
49	-4594920.198	0.9	2701382.187	0.6	-3492436.734	0.6
50	-4594972.889	1.0	2701338.316	0.5	-3492408.204	0.7

$v^T P v = 17.18$, $n-u+d = 3$, $VF = 5.7$ The standard deviations of the coordinates are smaller than from the minimally constrained solution for this same data as shown in Harvey (2009). The residuals and the VF are the same.

7. Summary, conclusions etc..

Write your own and submit them to the author.

8. Tutorial Questions

Q1. Datum defects - levelling.

Simulate an adjustment of Δh around a levelling network containing 7 points. Do not hold any point fixed. All observations are equal precision ($\pm 1\text{cm}$).

Observed height differences: 1-7, 1-6, 1-5, 1-4, 1-3, 1-2, 2-7, 2-6, 2-5, 2-4, 2-3, 3-7, 3-6, 3-5, 3-4, 4-7, 4-6, 4-5, 5-7, 5-6, 6-7.

Parameters: $h_1, h_2, h_3, h_4, h_5, h_6, h_7$

You can easily form the A matrix, $P = I$ in units of cm^{-2} and:

$$A^T P A = N = \begin{matrix} 6 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & 6 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & 6 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & 6 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 6 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & 6 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 6 \end{matrix}$$

- Try to calculate N^{-1} .
- Solve N^{-1} with h_7 fixed.
- Simulate a freenet solution of the Δh network.

Q2. Use Chifley Dam traverse data from these notes, and a supplied spreadsheet with A already calculated. Do the Freenet calculations in MS Excel or similar and show one more decimal place than in these notes for adjusted coordinate values.

Q3. In section 2 we looked at the effect of constraints on the point error ellipses. Now consider a traverse that has at least several legs of similar length and is 'straightish' rather than a loop. Let's say we have 15 points. Do some simulations yourself and look at the error ellipses for these options: a) points 1 and 2 fixed, b) points 7 and 8 fixed, c) points 1 and 15 fixed, d) a freenet solution.

Q4. Repeat the calculations shown in the examples in these notes using some other data e.g. the Oatley network from Harvey (2009).

9. References

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